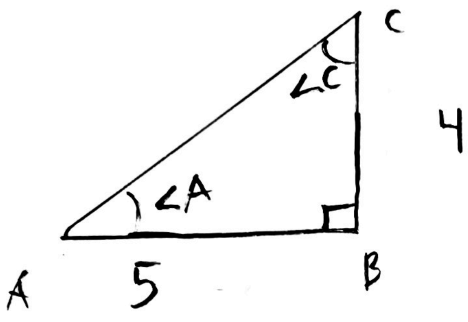
21.11. 22



Pytagoras

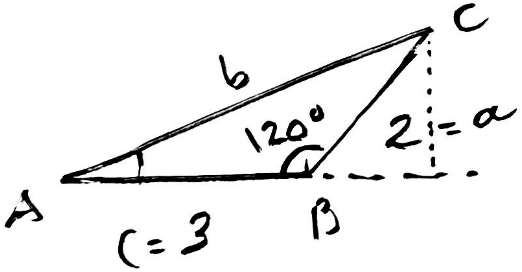
$|AC|^2 = 4^2 + 5^2 = 16 + 25 = 41$

$|AC| = \sqrt{41} \sim 6.403$

$\tan \angle A = \frac{4}{5} = 0.8$

$\angle A = \arctan(0.8) = \tan^{-1}(0.8) = 38.66°$

$\angle C = 90° - \angle A = 51.34°$



cosinussetning.

$b^2 = a^2 + c^2 - 2ac \cdot \cos(\angle B)$

$b^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \underbrace{\cos(120°)}_{-\sin(30°) = -\frac{1}{2}}$

$b^2 = 4 + 9 - 2 \cdot 2 \cdot 3 \cdot (-\frac{1}{2}) = 4 + 9 + 6 = 19$

$b = \sqrt{19} \sim \underline{4.359}$

Sinussetningen $\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$

$$\frac{\sin\angle A}{2} = \frac{\sin(120^\circ)}{\sqrt{19}} = \frac{\sin\angle C}{3}$$

$$\sin(\angle A) = \sin(120^\circ)\cdot\frac{2}{\sqrt{19}} = \sin(60^\circ)\cdot\frac{2}{\sqrt{19}}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{19}} = \sqrt{\frac{3}{19}}$$

(andre muligheter ~157° går ikke)

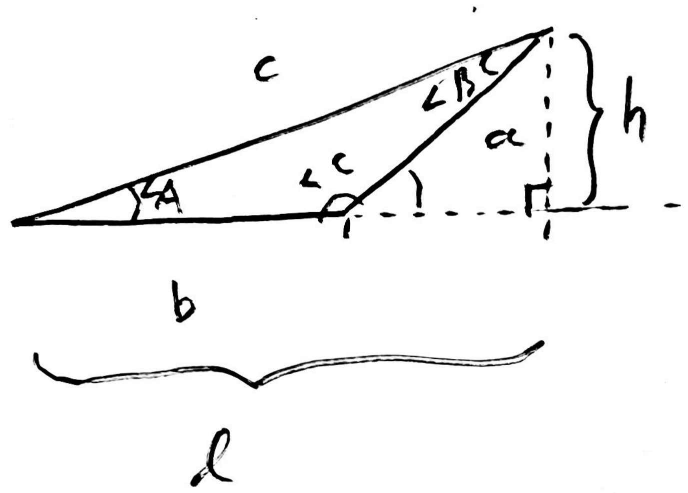
$$\angle A = \arcsin\left(\sqrt{\frac{3}{19}}\right) \approx \underline{23.41^\circ}$$

$$\angle C = 180^\circ - \angle A - \angle B = \underbrace{(180^\circ - 120^\circ)}_{60^\circ} - 23.41^\circ$$

$$= \underline{36.59^\circ}$$

Arealet til trekanten $\frac{1}{2}(3\cdot 2\cdot\sin(\overset{120^\circ}{60^\circ}))= 3\cdot\frac{\sqrt{3}}{2}$

Arealet $= \frac{3\sqrt{3}}{2} \approx \underline{2.598}$



Benytter Pytagoras på den «store rettvinkla trekanten»

$$h = a \cdot \sin(180^\circ - \angle C)$$
$$= a \sin(\angle C)$$

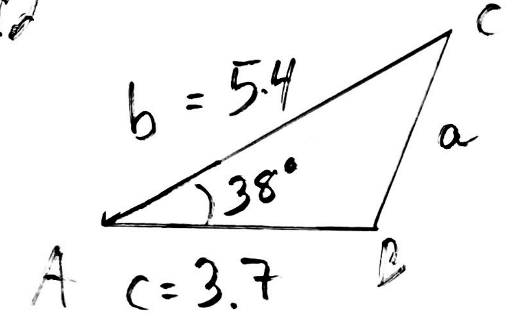
$$\ell = b - a\cos(\angle C)$$

$$c^2 = (a \sin \angle C)^2 + (b - a\cos \angle C)^2$$
$$= a^2 \sin^2(\angle C) + b^2 + a^2 \cos^2 \angle C - 2ab\cos(\angle C)$$

$$c^2 = a^2(\underbrace{\sin^2(\angle C) + \cos^2(\angle C)}_{1}) + b^2 - 2ab\cos(\angle C)$$

$$= a^2 + b^2 - 2ab\cos(\angle C)$$

Oppg



Finn a

$\angle B$ og $\angle C$

Areal.

Cosinussetning

$$a^2 = b^2 + c^2 - 2bc\cos(38°)$$

$$a = 3.3706$$

Sinussetningen

$$\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b}$$

$$\sin(\angle B) = \frac{b}{a}\sin(\angle A) \sim 0.9863\ldots$$

$$\angle B = 80.52° \text{ og } 99.48°$$

$a^2 + c^2 = 25.05 < b^2 \approx 29$ så $\angle B$ må vere $> 90°$

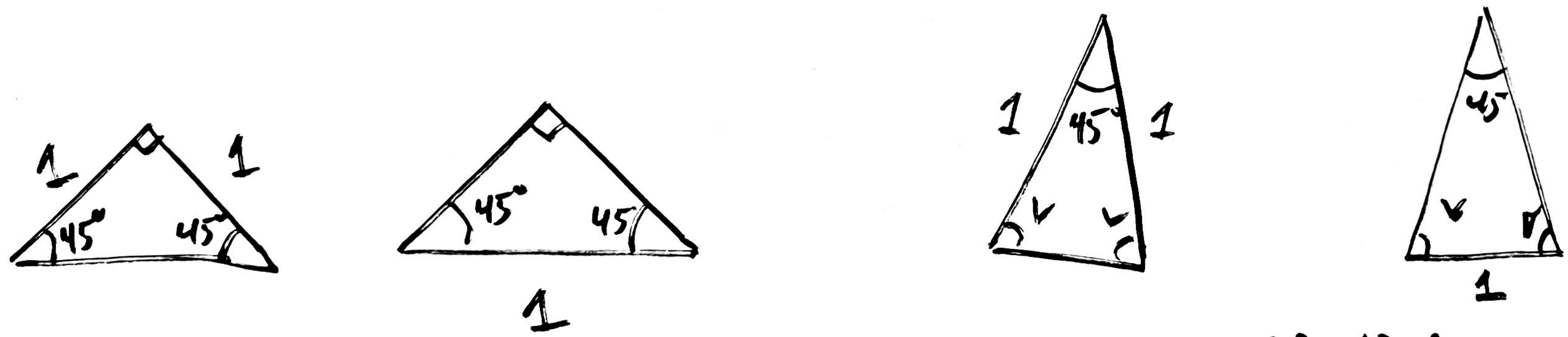
Derfor er $\underline{\angle B = 99.48°}$

Bruk av cos setning $\angle B$... gir entydig verdi til $\angle B$.

$$\angle C = 180^\circ - \underset{\angle A}{38^\circ} - \underbrace{99.48^\circ}_{\angle B} = \underline{42.52^\circ}$$

Arealet $\frac{1}{2} bc \cdot \sin(38^\circ) = \underline{6.15}$

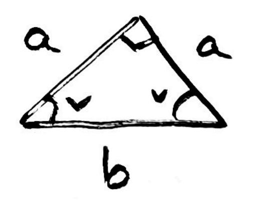
Oppg. Finn alle likebeina trekanter (minst 2 sider er like lange) hvor én vinkel er 45° og ene siden har lengde 1



$2v = 180^\circ - 45^\circ = 135^\circ$

$v = 135^\circ/2 = 67{,}5^\circ$

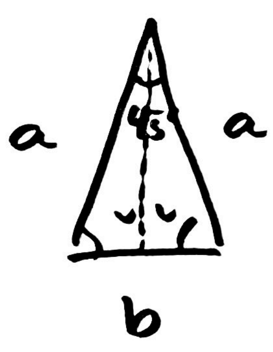
$V = 45°$



1. $a = 1$ ved Pyt. er $b^2 = a^2 + a^2 = 2a^2$

så $b = \sqrt{2}$

2. $b = 1$ $2a^2 = b^2 = 1$ så $a = \frac{1}{\sqrt{2}}$.



$V = \frac{135°}{2} = \underline{67.5°}$

$\frac{b}{2} = a \cdot \sin\left(\frac{45°}{2}\right)$

$b = a \cdot \underbrace{2\sin(22.5°)}_{0.76536\ldots}$

3. $a = 1$ da er $b = \underline{0.76536}$

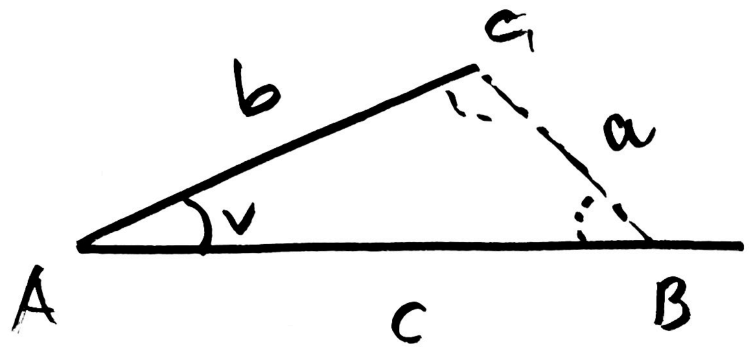
4. $b = 1$ da er $a = 1.3065$

Benytter sinussetning

$\frac{\sin V}{a} = \frac{\sin 45°}{b}$

$b = a \frac{\sin 45°}{\sin V}$

etc.

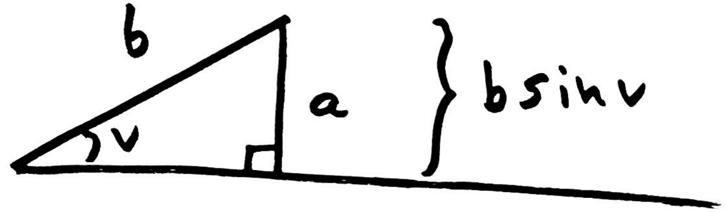


a, b og V gitt.

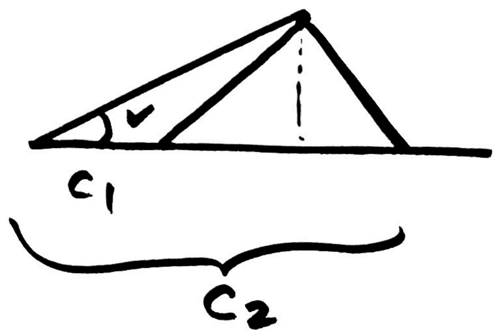
Hva er c og $\angle B$
$\angle C$.

$a < b\sin v$ ingen løsning

$a = b\sin v$ ēn løsning
rettvinkla Δ

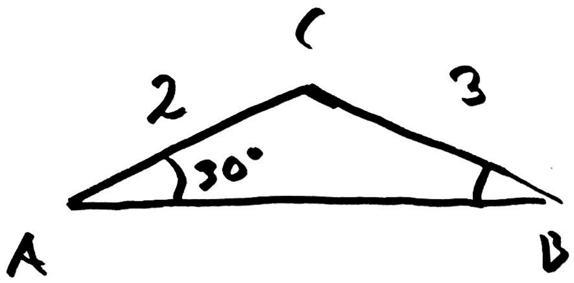


$b\sin v < a < b$ to løsninger



ēn løsning

$a > b$



hva er $\angle B$?

$$\frac{\sin 30^\circ}{3} = \frac{\sin \angle B}{2}$$

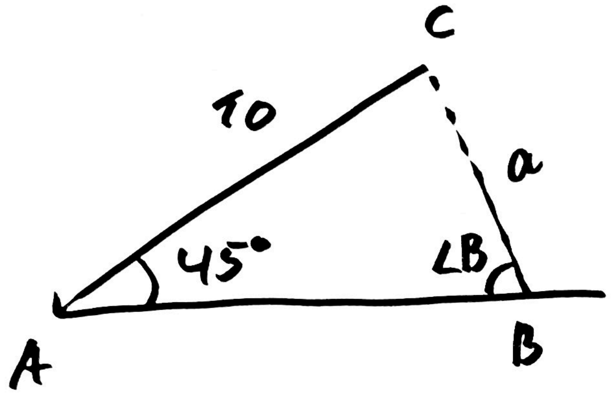
$$\sin(\angle B) = \frac{2}{3}\sin(30^\circ) = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$$

$$\angle B = \underline{19.5^\circ} \quad (\text{og } 160.5^\circ)$$

ikke mulig

$\angle A = 30^\circ$, $\angle B = 19.5^\circ$

$$\angle C = 180^\circ - 30^\circ - 19.5^\circ = \underline{130.5^\circ}$$



Finn $\angle B$

når 1) $a = 9$

2) $a = 2$

3) $a = 5\sqrt{2}$

4) $a = 20$

1. $\frac{\sin \angle B}{10} = \frac{\sin(45°)}{9} \Rightarrow \sin(\angle B) = \frac{10}{9} \cdot \overbrace{\sin(45°)}^{1/\sqrt{2}}$

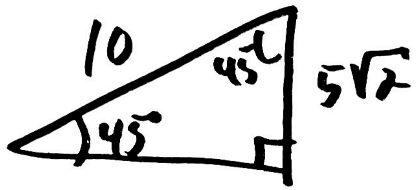
$\angle B_1 = \arcsin\left(\frac{10}{9}\frac{1}{\sqrt{2}}\right) = \arcsin(0.78567\ldots)$

$= 51.78°$

$\angle B_2 = 180° - \angle B_1 = 128.22°$

2) $a = 2 < 10 \cdot \sin 45^\circ = 10 \cdot \frac{1}{\sqrt{2}} = 5 \cdot 2/\sqrt{2} = \underline{5\sqrt{2}}$

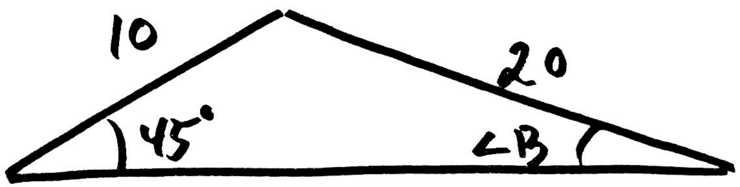
ingen løsning.

3) 

likebeinet trekant

$\angle B = 90^\circ$

4) $a = 20$



$\frac{\sin 45^\circ}{20} = \frac{\sin \angle B}{10}$ så $\sin(\angle B) = \left(\frac{1}{\sqrt{2}}\right) \cdot \frac{10}{20}$

$= \frac{1}{2\sqrt{2}}$.

$\angle B = \arcsin\left(\frac{1}{2\sqrt{2}}\right) \approx \underline{20.7^\circ}$