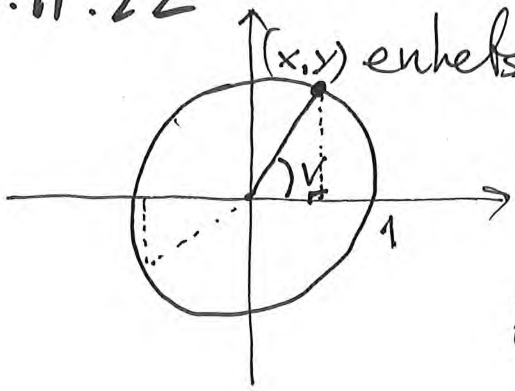
16.11.22



enhetssirkelen

$x = \cos v$

$y = \sin v$

$\tan v = \frac{\sin v}{\cos v}$

Stigningstallet til linjen gjennom origo og (x,y).

Generelt $\sin(90° - v) = \cos(v)$

$\cos(90° - v) = \sin(v)$

(Refleksjon om linjen $x = y$)

Pytagoras : $(|\sin v|)^2 + (|\cos v|)^2 = 1^2$

$$\boxed{\sin^2 v + \cos^2 v = 1}$$

alle v.

invers funksjoner

$\arcsin(y) = \sin^{-1}(y)$

er vinkelen mellom $-90°$ og $90°$

slik at $\sin(\sin^{-1}(y)) = y$

$\arccos x = \cos^{-1}(x)$

er vinkelen mellom $0°$ og $180°$

slik at $\cos(\cos^{-1}(x)) = x$

Merk $\sin^2 v = (\sin v)^2$

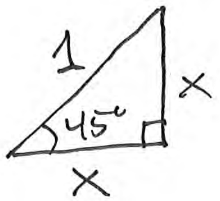
$\sin v^2 = \sin(v^2)$

men $\sin^{-1} v = \arcsin v$

ikkje $\frac{1}{\sin v} = (\sin v)^{-1}$!.

Eksakte verdier for trig. funksjoner.

$\underline{45^\circ}$



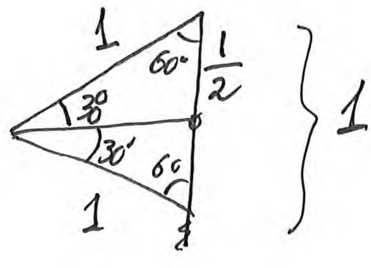
Pytagoras $x^2 + x^2 = 1^2$

$2 \cdot x^2 = 1$

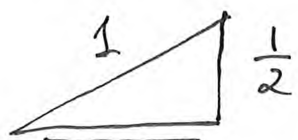
$x = \frac{1}{\sqrt{2}}$ $(x > 0)$

$$\cos(45^\circ) = \sin(45^\circ) = \frac{1}{\sqrt{2}} \sim 0.707$$

$$\tan(45^\circ) = 1$$



Pytagoras



$\sqrt{1 - \left(\frac{1}{2}\right)^2}$

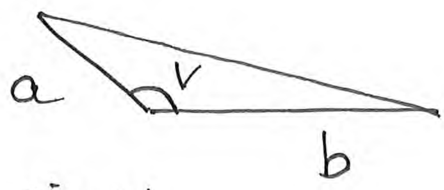
$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \sim 0.866..$

$$\sin(30^\circ) = \cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}.$$

$$\tan(60^\circ) = \frac{\sin(60^\circ)}{\cos(60^\circ)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \sim 1.73$$

Arealsetningen



Arealet $A = \underline{\frac{a \cdot b}{2} \cdot \sin v}$

oppg. Δ to av sidene har lengde 2 og 3.
Hvis arealet til Δ er 2, hva er da vinkelen mellom sidene med lengde 2 og 3?

$a = 2, \quad b = 3 \qquad A = 10$

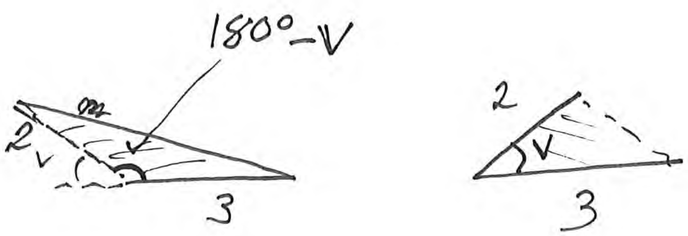
hva er v?

$$2 = \frac{2 \cdot 3}{2} \sin v$$

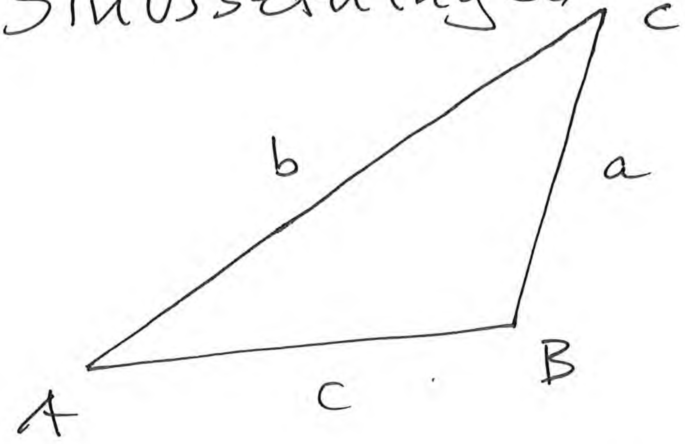
$$\sin v = \frac{2}{3} \sim 0.66$$

$$v = \arcsin\left(\tfrac{2}{3}\right) \sim 41.81^\circ$$

og $\quad 180^\circ - \arcsin\left(\tfrac{2}{3}\right) = \underline{138.19^\circ}$



Sinussetningen

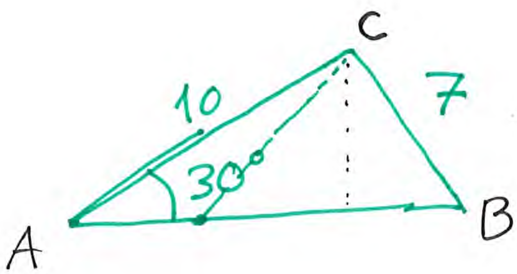


$$\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b} = \frac{\sin(\angle C)}{c}$$

$$\frac{2\cdot \text{Areal}}{a\cdot b\cdot c} = \frac{2(\frac{1}{\cancel{2}}\cdot c\cdot b\cdot \sin\angle A)}{abc} = \frac{\sin(\angle A)}{a}$$

tilsvarende ved bruk av arealsetning og $\angle B$ og $\angle C$.

Eksempel



Benytter sinussetning.

$$\frac{\sin(\angle A)}{a} = \frac{1/2}{7} = \frac{1}{14}$$

så $\frac{1}{14} = \frac{\sin(\angle B)}{10} = \frac{\sin\angle C}{c}$

$$\sin(\angle B) = \frac{10}{14} = \frac{5}{7}$$

$\angle B_1 = 45.58°$

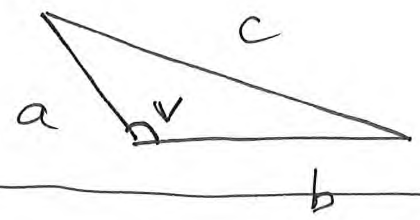
$\angle B_2 = 134.42°$

$$\angle C_1 = 180° - 30° - \angle B_1$$
$$= 150° - 45.58°$$

$$\angle C_1 = \underline{104.42°}$$
$$\angle C_2 = 150° - 134.42° = \underline{15.58°}.$$

$$c = 14 \sin(\angle C)$$
$$c_1 = 14 \sin(\angle C_1) \sim \underline{13.56}$$
$$c_2 = 14 \sin(\angle C_2) \sim \underline{3.76}$$

Cosinussetningen

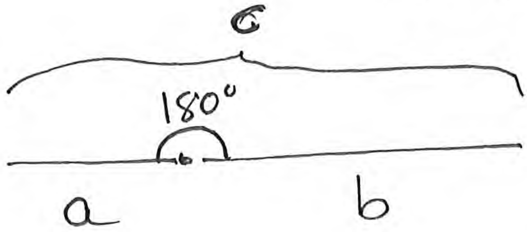


$$\boxed{c^2 = a^2 + b^2 - 2ab\cos v}$$

$v = 90°$ $\quad \cos v = 0$ $\quad c^2 = a^2 + b^2$

Pytagoras

$v = 180°$



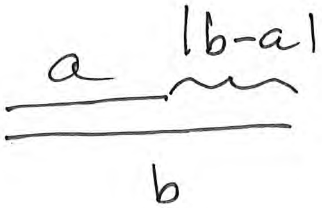
$\cos(180°) = -1$

$$c^2 = a^2 + b^2 - 2ab(-1)$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

kvadratsetning

$v = 0°$

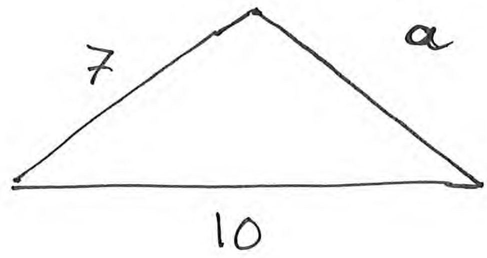


$\cos(0°) = 1$

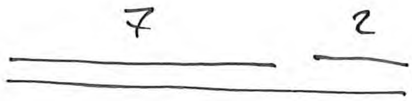
$$c^2 = a^2 + b^2 - 2ab \cdot 1$$

$$(b-a)^2 = a^2 + b^2 - 2ab$$

Eksempel



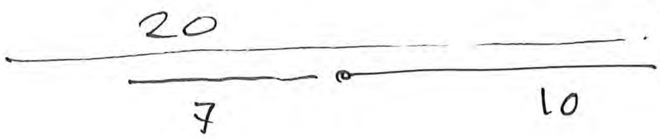
$a = 2$



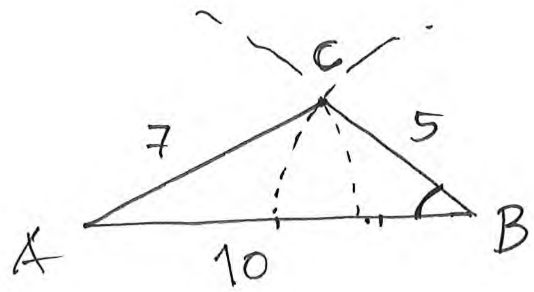
Finnes ingen slik trekant

— || —

$a = 20$



$a = 5$



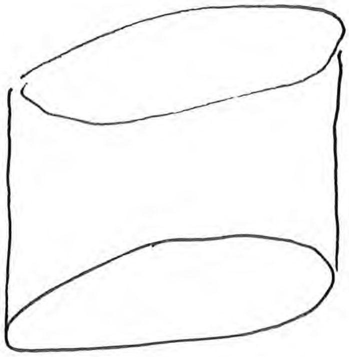
$$7^2 = 10^2 + 5^2 - 2 \cdot 5 \cdot 10 \, \underline{\cos(\angle B)}$$

$$49 = 100 + 25 - 100 \cos(\angle B)$$

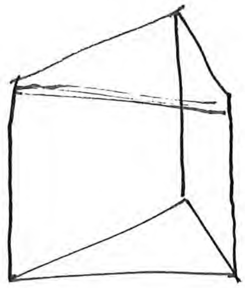
$$-76 = -100 \cos(\angle B)$$

$$\cos(\angle B) = \frac{76}{100} = 0{,}76$$

$$\angle B = \arccos(0.76) = \underline{40.53\ldots^\circ}$$



Prisme

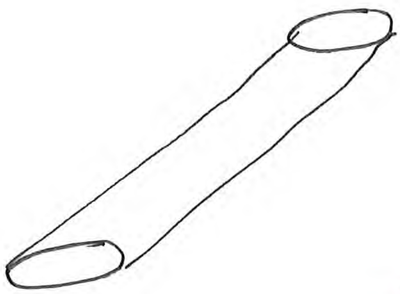


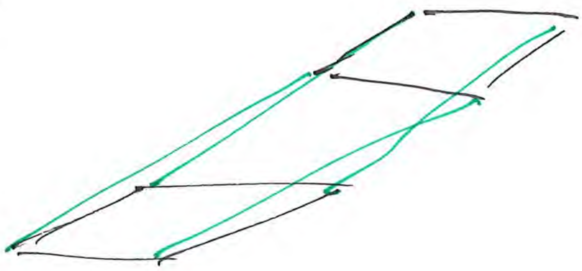
Sylinder

sirkel

$V = \underline{\pi r^2 h}$

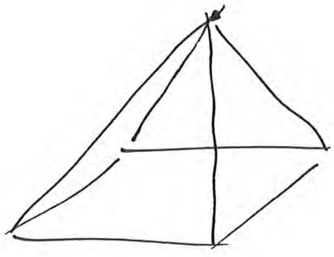
Volum = areal til grunnflate × høyde.

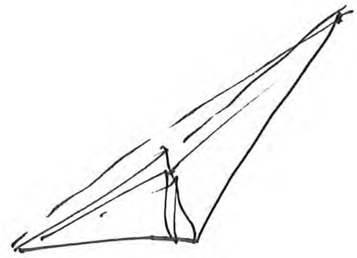


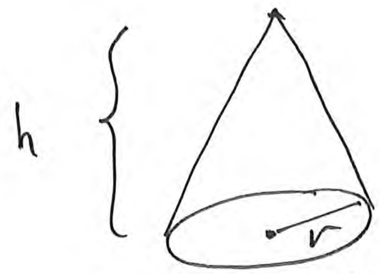


skjevt prisme

Pyramider kjegler og kuler





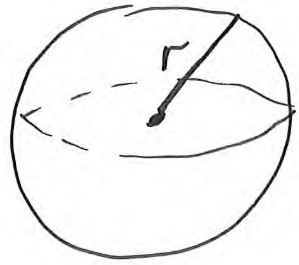


$V = \frac{1}{3} h \cdot \pi r^2$

skeiv pyramide

$$V = \frac{1}{3}(\text{areal til grunnflate}) \cdot \text{høyde}$$

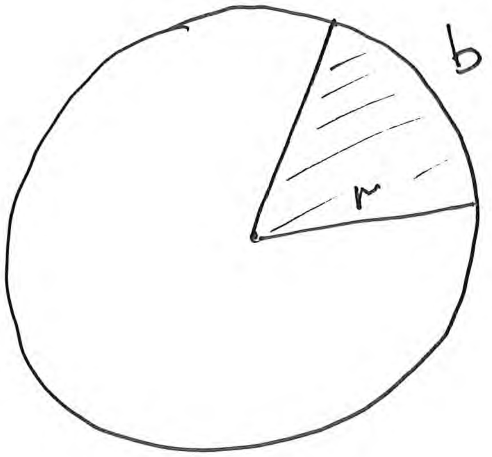
Volum kule:



$$\underline{V = \frac{4\pi r^3}{3}} \approx 4.1887\, r^3$$

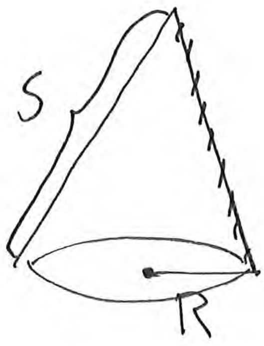
overflatearealet til en kule med radius r:

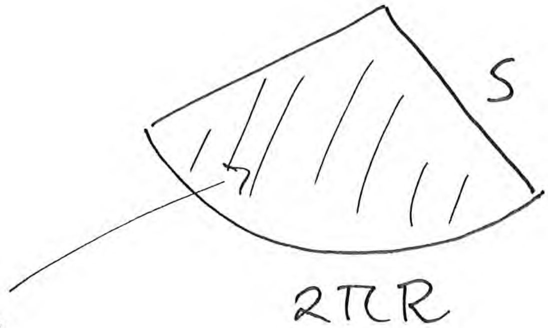
$$\left(\frac{4\pi r^3}{3}\right)' = \underline{4\pi r^2}$$



arealet til
sirkelsegmentet

$$A = \frac{r \cdot b}{2}$$





Arealet

$$\frac{1}{2}(2\pi R)\cdot s = \underline{\pi R s}$$