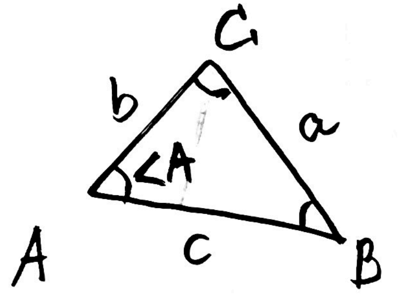
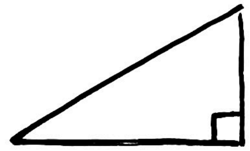
14. nov 22

Trigonometri

Kap 9

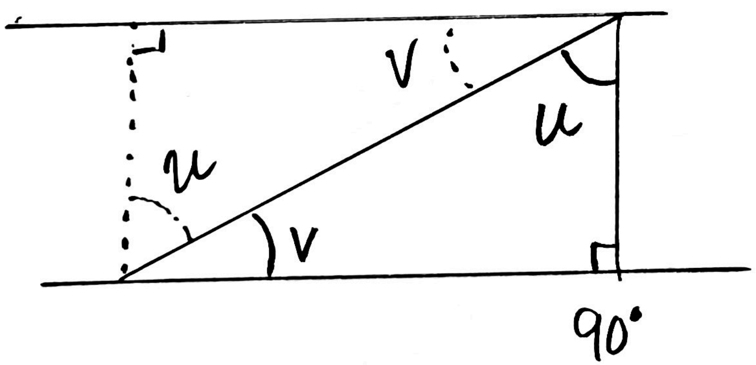


∠A vinkel A.



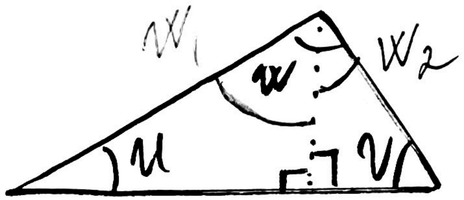
Rettvinkla trekant

$$\boxed{\text{Summen av vinklene i enhver trekant er } 180°}$$



Så $u + v = 90°$

Summen av vinklene i en rettvinkla trekant er $u + v + 90° = 90° + 90° = \underline{180°}$



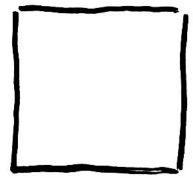
$w_1 + w_2 = w$

$u + w_1 = 90°$

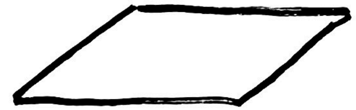
$v + w_2 = 90°$ så $\underline{u + v + w = 180°}$

Firkant

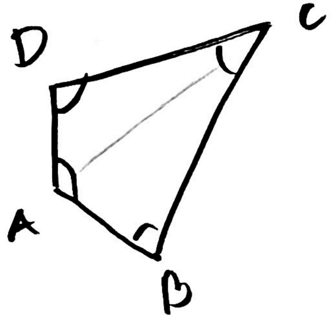
Rektangel



Kvadrat

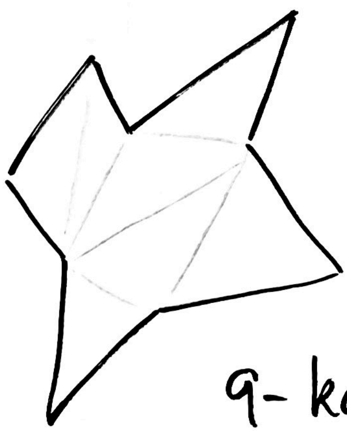


parallellogram



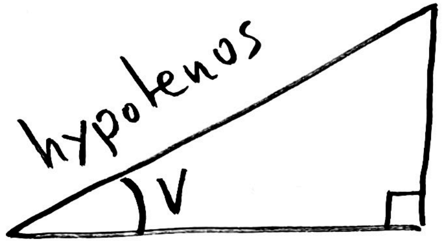
$\angle A + \angle B + \angle C + \angle D = 180° + 180° = 360°$

n-kant

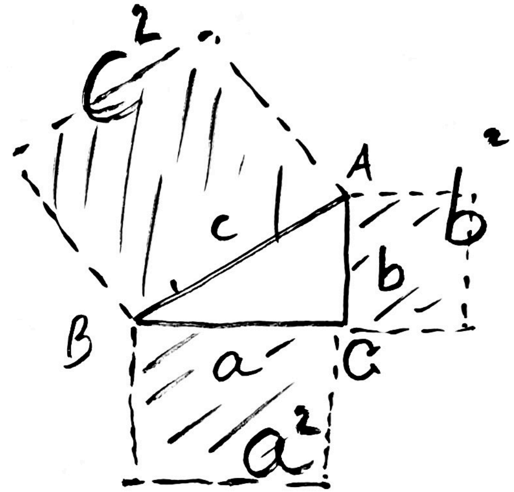


9-kant

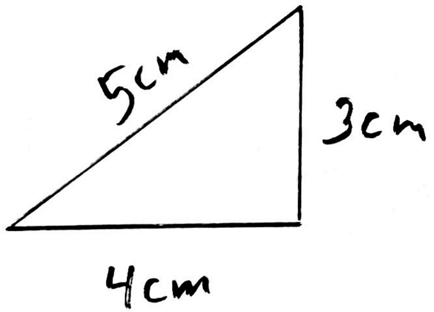
Summen av vinklene i en n-kant er $(n-2) \cdot 180°$



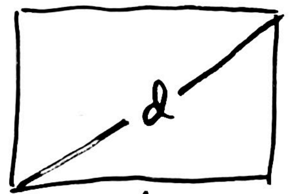
Pytagoras sin sats



$c^2 = a^2 + b^2.$



$3^2 + 4^2 = 9 + 16 = 25 = 5^2$



skjermstørrelse er gitt ved d.

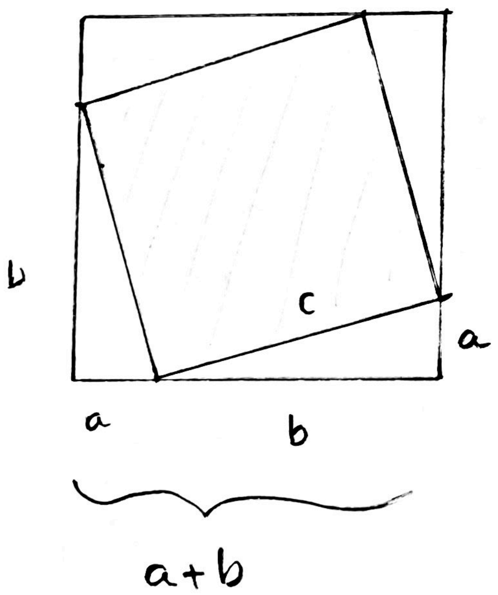
Skjerm med forhold 4:3

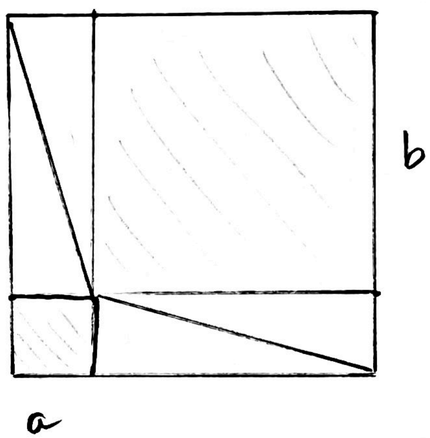
bredden $= \frac{4}{5} \cdot d$ høyden $= \frac{3}{5} \cdot d$

$d = 25''$ da er bredden $20''$

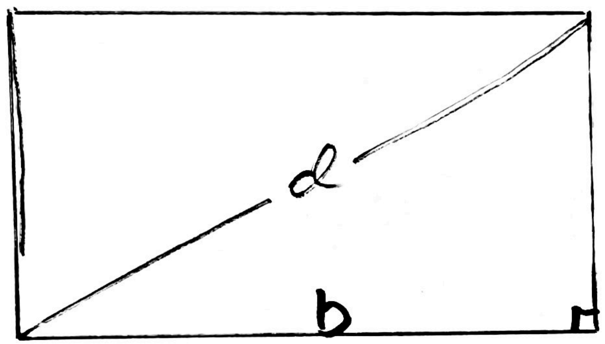
høyden $15''$.

Arealet bredde · høyde : $\frac{4 \cdot 3}{5^2} d^2 = \underline{\frac{12}{25} d^2}$





Eksempel



16 : 9

Hva er bredden og høyden uttrykt ved d.

$$\frac{b}{h} = \frac{16}{9}.$$

$$b^2 + h^2 = d^2$$

$$b = \frac{16}{9} h$$ så $$\left(\frac{16}{9}\right)^2 h^2 + h^2 = d^2$$

$$\left(\left(\frac{16}{9}\right)^2 + 1\right) h^2 = d^2 \qquad h^2 = d^2 \frac{1}{1+(16/9)^2}$$

$$h = \frac{d}{\sqrt{1+(16/9)^2}} = \frac{d}{\sqrt{1+(4/3)^4}} \sim 0.49 \cdot d$$

tilsvarende $$b = \frac{d}{\sqrt{1+(9/16)^2}} \sim \underline{0.87 \cdot d}$$

Pytagoreiske trippel er heltall (a, b, c)

slik at $a^2 + b^2 = c^2$

eks: $3^2 + 4^2 = 5^2$

$5^2 + 12^2 = 13^2$

$25 + 144 = 169$

Det finnes uendelig mange slike Pyt. trippel.

$a, b \in \mathbb{N}$

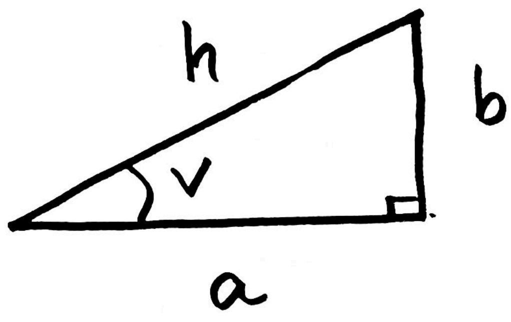
$$(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$$

$$(a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$$

$$= (a^2 - b^2)^2 + 4a^2b^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$\underline{(a^2 + b^2)^2 = (2ab)^2 + (a^2 - b^2)^2}$$

Sinus og cosinus



$$\sin(v) = \frac{\text{motstående katet}}{\text{hypotenus}} = \frac{b}{h}$$

$$\text{cosin}(v) = \frac{\text{hosliggende katet}}{\text{hypotenus}} = \frac{a}{h}$$

$v = 0°$: $\sin(0°) = 0$
$\text{cosin}(0°) = 1$

$v = 90°$ $\sin(90°) = 1$
$\cos(90°) = 0$

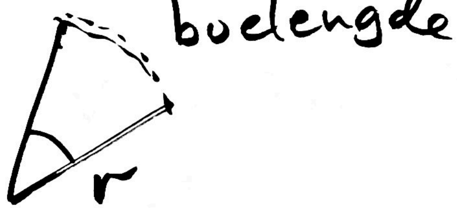
oppg Regn ut

$\sin(25°) = 0.4226...$

$\cos(65°) = 0.4226...$

Enheter for vinkler

	$\boxed{\text{deg}}$	90°	$\underline{360^\circ}$	$\underline{\text{grader}}$ 30°
Gradianer Gon	$\boxed{\text{gra}}$	100	$\underline{400}$	gradianer
Radianer	$\boxed{\text{rad}}$	$\pi/2 \sim 1.57\ldots$	$\underline{2\pi}$	radianer.



$$\frac{\text{buelengde}}{\text{radi}}$$

inverssinus funksjonen

$\sin^{-1} a$ er en vinkel mellom $-90°$ og $90°$

slik at $\sin(\sin^{-1} a) = a$

for $a \in [-1, 1]$

$\sin^{-1}(0.5) \neq \frac{1}{\sin(0.5)}$

$\sin^{-1}$ er inversfunksjon

$\arcsin(V) = \sin^{-1}(V)$

"arcus sinus"

$\sin^{-1}(-0.5) = -30°$

$\sin^{-1}(0.5) = 30°$

opg

$\sin^{-1}(0.7) = 44.427...$

inverscosinus

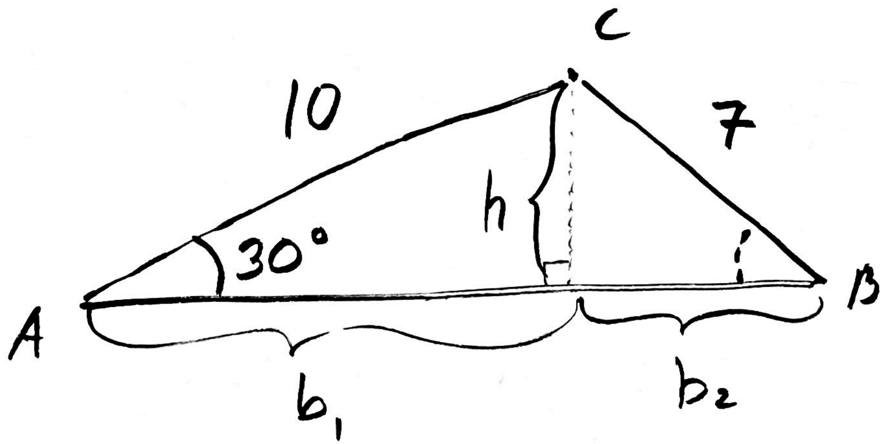
$0^\circ \leq \cos^{-1} a \leq 180^\circ$ $\qquad a \in [-1, 1]$.

$\cos(\cos^{-1}(a)) = a$

$\arccos = \cos^{-1}$

$\angle C > 90^\circ$

Eks



$\frac{h}{10} = \sin 30^\circ = \frac{1}{2}$ så $h = 10\sin(30^\circ) = 5$

$\frac{b_1}{10} = \cos(30^\circ)$ så $b_1 = 10 \cdot \cos(30^\circ) = 8.66$

$$\sin(\angle B) = \frac{h}{7} = \frac{5}{7}$$

$$\angle B = \sin^{-1}\left(\frac{5}{7}\right) = \arcsin\left(\frac{5}{7}\right) = 45.58\ldots^\circ$$

(merk: $\frac{5}{7} \cdot \frac{7}{7} = \frac{5 \cdot 7}{49} \sim \frac{7}{49/5} \sim 0.7$)

alternativt ved Pyt

$$b_2 = 7 \cdot \cos(\angle B)$$

$$b_2^2 + h^2 = 7^2$$

$$b_2 = \sqrt{7^2 - 5^2} = \sqrt{49-25} = \sqrt{24}$$

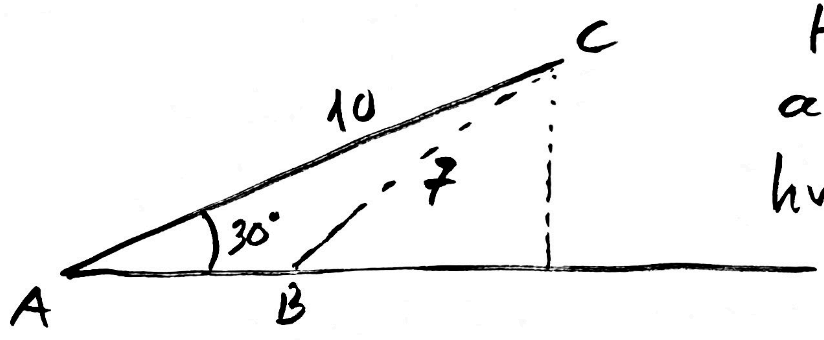
$$= 2\sqrt{6} \sim 4.8989\ldots$$

Side c er $b_1 + b_2 = 10\cos 30^\circ + 7 \cdot \cos(\angle B)$

$$\sim \underline{13.56}$$

Vinkel C: $\angle C = 180^\circ - \angle A - \angle B$

$$= 150^\circ - 45.58^\circ = \underline{104.42^\circ}$$

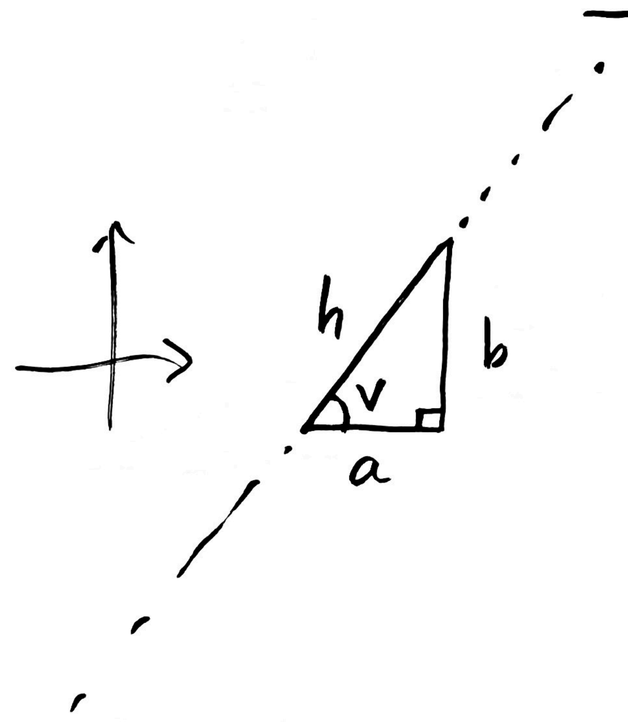


Her er en annen mulighet

hva $\angle C < 90°$

$\angle B > 90°$

Finn gjerne vinklene og lengden på AB i dette tilfellet.



Tangens

$$\tan(V) = \frac{b}{a} = \text{stigningstallet}$$

$a \neq 0$

$$\tan(V) = \frac{b}{a} = \frac{b/h}{a/h} = \frac{\sin V}{\cos V}$$

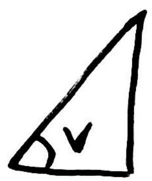
$$\boxed{\tan V = \frac{\sin V}{\cos V}}$$

inverstangens $\tan^{-1}(s)$ $s \in \mathbb{R}$

$= \arctan(s)$

vinkelen i $\langle -90°, 90° \rangle$

slik at $\tan(\arctan(s)) = s.$



$\tan(v) = 2$

så $V = \arctan(2) = \underline{63.4°}$

Stigning på 30%.

Stigningstallet $s = 30\% = 0.3$

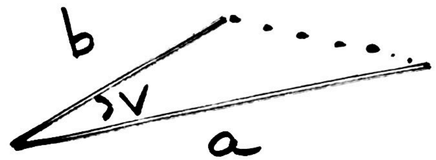
Vinkelen

$V = \arctan(0.3) = \underline{16.7°}$

$V = 89°$ Stigningstallet er da $\tan(89°) = \underline{57.3}\ldots$

$80°$ ——— $\tan(80°) = \underline{5.67}\ldots$

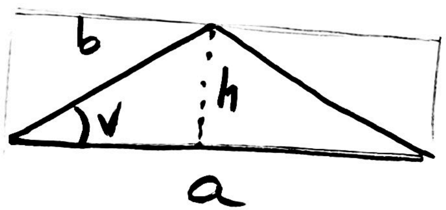
Arealsetning



Arealet til Δ er lik

$$\underline{A = \frac{1}{2} a \cdot b \cdot \sin(V)}$$

$0 \leq V \leq 90°$



$h = b\sin(v)$

Arealet til Δ er

$\frac{1}{2} \cdot$ bredde $\cdot$ høyde

$\frac{1}{2} a \cdot b \cdot \sin v = \underline{\frac{ab}{2} \sin(v)}$