7.11.2022.

e Eulertallet 2,7182

$$(e^x)' = e^x$$

$a > 0$

$$a^x = (e^{\ln a})^x = e^{x \cdot \ln a}$$

$$(a^x)' = (\ln a) \cdot a^x$$

$\ln = \log_e$.

$$(\ln x)' = \frac{1}{x}$$
$x > 0$

$$(\ln(-x))' = \frac{-1}{-x} = \frac{1}{x}$$
$x < 0$

$$(\ln|x|)' = \frac{1}{x}$$

Eks

$$(x \cdot \ln x)' = (x)' \ln x + x \cdot (\ln x)'$$
$$1 \cdot \ln x + x \cdot \frac{1}{x}$$
$$= \underline{\ln x + 1}$$

Siden $1 = x'$

$$(x \cdot \ln x)' = \ln x + (x)'$$

$$\underline{(x \cdot \ln x - x)' = \ln x}$$

- Deriver $\ln(x^4+5) = \ln(u(x))$ hvor kjernen er $u(x) = x^4+5$

$$(\ln(x^4+5))' = \frac{1}{x^4+5} \cdot (x^4+5)' \quad \left(= \frac{1}{u} \cdot u'\right)$$

$$= \frac{1}{x^4+5}(4x^3) = \underline{\frac{4x^3}{x^4+5}}$$

oppg Derivere $x^3 \ln(-4x)$ (def. for $x < 0$)

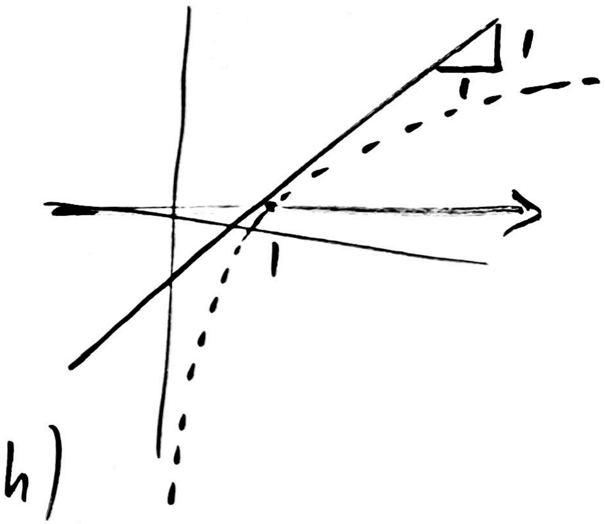
$$(x^3 \ln(-4x))' = (x^3)' \ln(-4x) + x^3 (\ln(-4x))'$$

$$= 3x^2 \ln(-4x) + x^3 \frac{(-4x)'}{-4x}$$

$$= 3x^2 \ln(-4x) + x^3 \cdot \frac{1}{x} = \underline{x^2(3\ln(-4x) + 1)}$$

$$\left(\ln(u(x))\right)' = \frac{1}{u(x)} \cdot u'(x) = \underline{\frac{u'(x)}{u(x)}}$$

$$\frac{d}{dx}\ln(u(x)) = \frac{d\ln u}{du} \cdot \frac{du}{dx} = \frac{1}{u(x)} \cdot u'(x)$$



$$\frac{d}{dx}\ln(x)\Big|_{x=1} = \frac{1}{x}\Big|_{x=1} = 1$$

Fra definisjonen er dette lik

$$\lim_{h\to 0} \frac{\ln(1+h) - \ln 1}{h} = \lim_{h\to 0} \frac{1}{h}\ln(1+h)$$

$$= \lim_{h\to 0} \ln\left((1+h)^{1/h}\right) = \ln\left(\lim_{h\to 0}(1+h)^{1/h}\right) = 1 = \ln e.$$

Derfor er $e = \lim_{h \to 0} (1+h)^{1/h}$ $\qquad h = \frac{1}{n}$, n naturlige tall

$$\boxed{e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n}$$

(rekker)

Faktum $\quad e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots$ $\qquad n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots n$

$= \underset{n \geq 0}{\text{summen av}} \ \frac{1}{n!}$ $\qquad 0! = 1$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{3n} = \lim_{n \to \infty} \left(\left(1 + \frac{1}{n}\right)^n\right)^3$$

$$= \left(\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n\right)^3 = (e)^3 = \underline{e^3}$$

$P(t)$ penger ved tiden t

$P_0 = P(0)$.

Årlig rente $r_{p.a}$

$r_{p.a} = 10\% = 0.1 = \frac{10}{100}$

$$P_1 = P_0 + r_{p.a.} P_0$$

$$= P_0(1 + r_{p.a})$$

$$P(2) = P_1(1 + r_{p.a}) = P_0(1 + r_{p.a})^2$$

$$P(n) = P_0(1 + r_{p.a})^n.$$

Kontinuerlig rente.

$$P_0\left(1 + \frac{rt}{n}\right)^n$$

Deler opp tidsintervalle i n like biter og legger til renten i slutten av hver intervall.

I grensen $n \to \infty$

$$P(t) = P_0 \lim_{n \to \infty}\left(1 + \frac{rt}{n}\right)^n$$

$t > 0$, $r > 0$

$m = \frac{n}{rt} \to \infty$ når $n \to \infty$.

$$P(t) = P_0 \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{m\cdot rt}$$

$$= P_0 \Big(\underbrace{\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{m}}_{e}\Big)^{rt}$$

$$\boxed{P(t) = P_0\, e^{rt}}$$

$r = 100\%$ kontinuerlige renter

$$P(1) = P_0\, e^{1\cdot 1} = e P_0$$
$$\sim \underline{2.7182\ldots\, P_0}$$

$$P(1) = (1+r)\cdot P_0 = \underline{2P_0}$$

$r_{p.a} = 100\%$ årlig rente

månedlig rente: $\left(1+\frac{1}{12}\right)^{12} P_0 = 2.613\ldots \cdot P_0$

daglig rente: $\left(1+\frac{1}{365}\right)^{365} P_0 = 2.714567\ldots\, P_0$

Sammenheng mellom kont. og årlig rente

$$\boxed{e^r = 1 + r_{p.a}}$$

$$P(t) = P_0 e^{rt} = \underline{P_0 (1 + r_{p.a})^t}$$

$$r = \ln(1 + r_{p.a})$$

$r_{p.a}$	r kont
2%	1.98%
10%	9.53%
100%	69.31%

$$P'(t) = P_0 (e^{rt})' = P_0 \cdot (rt)' e^{rt}$$

$$\underline{= r P(t)}$$

$$\frac{P'(t)}{P(t)} = r$$

Likninger.

$$\ln|x| + \ln|x+1| = 1$$

$$e^{\ln|x(x+1)|} = e^{\ln(e)} = e^{1}$$

$$|x(x+1)| = e^{1} = e$$

$$x(x+1) = \pm e$$

2.grads likning.

$a=1 \quad b=1 \quad c=\pm e$

$$x^2 + x \mp e = 0$$

benytter abc-formelen

$$x = \frac{-b \pm \sqrt{1^2 - 4\cdot 1(\mp e)}}{2} = \frac{-1 \pm \sqrt{1 \pm 4e}}{2}$$

ingen løsning for $x(x+1) = -e$

$x(x+1) = e$: $x = \dfrac{-1 \pm \sqrt{1+4e}}{2}$

Oppg

$$\ln|x| - \ln|x+1| = 1$$

$$\ln|x| + \ln(|x+1|^{-1}) = 1$$

$$\ln\left(|x| \cdot \frac{1}{|x+1|}\right) = \ln\left|\frac{x}{x+1}\right| = 1$$

$$e \qquad\qquad e$$

$$e$$

$$\left|\frac{x}{x+1}\right| = e.$$

så $\dfrac{x}{x+1} = \pm e \iff x = \pm e(x+1)$

$$= \pm e x \pm e$$

$$(1 - (\pm)e)x = \pm e$$

$$(1 + \mp e)x = \pm e$$

Løsningene er $x = \dfrac{\pm e}{1 + \mp e}$

det vil si $\underline{x = \dfrac{e}{1-e}}$ og $\underline{x = \dfrac{-e}{1+e}}$

Kromatisk skala.

Deler en oktav opp i n halvtoner.

Forholdet mellom etterfølgende halvtoner x,
samme for alle halvtoner

$x^n = 2$, en oktav opp

$\underline{x = \sqrt[n]{2}}$.

$1, (\sqrt[n]{2}), (\sqrt[n]{2})^2, \dots, 2$

Forhold mellom toner

kvint $3/2 = 1.5$

kvart $4/3 = 1.33\ldots$

ters $5/4 = 1.25$

(liten ters) $6/5 = 1.2$

Kurvedrøfting $f(x) = \frac{x}{\ln x}$ $\qquad D_f = \langle 0,1\rangle \langle 1,\infty\rangle$

$$f'(x) = \left(x \cdot \left(\frac{1}{\ln x}\right)\right)' = (x)' \cdot \frac{1}{\ln x} + x\left(\frac{1}{\ln x}\right)'$$

$$= 1 \cdot \frac{1}{\ln x} + x\left(\frac{-1}{(\ln x)^2}\underbrace{(\ln x)'}_{\frac{1}{x}}\right)$$

$$f'(x) = \frac{1}{\ln x} - \frac{1}{(\ln x)^2} = \underline{\frac{\ln x - 1}{(\ln x)^2}}$$

$f'(x) = 0$ når $\ln x = 1$ så $x = e^{\ln x} = e^1 = \underline{\underline{e}}$

$\ln x$ økende så $f'(x) < 0$ for $x < e$

$f'(x) > 0$ for $x > e$

<u>Bunnpunkt</u> i $(e, f(e)) = \underline{(e, e)}$

Vertikal asymptote i $x=1$

$f(x)$ synkende $\langle 0,1\rangle$ og $\langle 1,e\rangle$

stigende $\langle e,\infty\rangle$

$\ln x = u$

$$f''(x) = \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right)'$$

$$= \left(\frac{1}{u(x)} - \frac{1}{u^2(x)}\right)' = \left(u^{-1}(x) - u^{-2}(x)\right)'$$

$$f''(x) = \left(-u^{-2} - (-2)u^{-3}\right)\cdot u'$$

$$= \frac{1}{x}\left(\frac{-1}{(\ln x)^2} + \frac{2}{(\ln x)^3}\right) = \frac{(-\ln x + 2)}{x(\ln x)^3}$$

$f''(x) = 0$ når $\ln x = 2$ $\qquad \underline{x = e^2} \sim 7.3891$

$f''(x) < 0$ for $x \in \langle 0,1\rangle$ konkav ned.

$x > 1$ $\qquad f''(x) > 0$ for $x \in \langle 1, e^2\rangle$ konkav opp

$f''(x) < 0$ for $x \in \langle e^2, \infty\rangle$ konkav ned.

Vendepunkt i $(e^2, f(e^2)) = \underline{(e^2\ \frac{e^2}{2})}$.

$$p(n) = \overset{\text{antall}}{\#} \text{ primtall } \leq n$$

$$\frac{n/\ln(n)}{p(n)} \to 1 \quad \text{når } n \to \infty.$$

$$\log x = \frac{\ln x}{\ln(10)} = \frac{\ln x}{2.3026\ldots}$$

$$(\log x)' = \frac{1}{2.3026\ldots}(\ln x)' = \frac{1}{(2.3026\ldots)x}$$

generelt

$$\log_a x = \frac{\ln x}{\ln(a)}$$

$$(\log_a x)' = \underline{\frac{1}{\ln a} \cdot \frac{1}{x}}.$$

$e^x = 3x$ $\qquad$ $x = \ln e^x = \ln(3x)$

$= \ln 3 + \ln x$?

Må ty til numeriske løsninger

$x \sim 0.6191$ og $x \sim 1.5121$

Tegna opp $(e^x - 3x)$ i geogebra og leser av nullpunktene.