4 nov 22

Deriver 1) $f(x) = \frac{-2}{x} - 3x\sqrt{x}$

2) $g(x) = \ln\left(\frac{x^2}{e^{3x+5}}\right)$

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$f(x) = -2 \cdot x^{-1} - 3x^1 x^{1/2} = -2x^{-1} - 3x^{3/2}$

$f'(x) = -2(x^{-1})' + (-3)(x^{3/2})'$

$= -2(-1 \cdot x^{-2}) - 3\left(\frac{3}{2} x^{1/2}\right)$

$= \frac{2}{x^2} - \frac{9}{2}\sqrt{x}$

$\ln x^r = r \ln x$

2) $g(x) = \ln\left(x^2 \cdot (e^{3x+5})^{-1}\right) = \ln(x^2) - \ln(e^{3x+5})$

$= 2\ln x - (3x+5)$ så $\underline{g'(x) = \frac{2}{x} - 3}$

Løs likningen $(\log x)^2 - \underbrace{\log(x^3)}_{3\log x} = 4$

$\log = \log_{10}$

$\ln = \log e$

$\log x = u$

$x = 10^{\log x} = 10^u$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4$ og $u = -1$

så $x = 10^4 = \underline{10\,000}$ og $x = 10^{-1} = \underline{\frac{1}{10}}$

Deriver $h(x) = \frac{x^2}{e^x} = x^2 \cdot (e^x)^{-1}$

$= x^2 e^{-x}$

$(e^x)' = e^x$.

$h'(x) = (x^2)' e^{-x} + x^2 (e^{-x})' = 2x e^{-x} + x^2 (e^{-x} \underbrace{(-x)'}_{-1})$

$= 2x e^{-x} - x^2 e^{-x} = (2x - x^2) e^{-x} = (2x - x^2)/e^x$

$= \underline{\frac{2x - x^2}{e^x}}$

$$(a^x)' = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = a^x \underbrace{\lim_{h\to 0} \frac{a^h - 1}{h}}_{\text{lik 1 når } a = e}$$

(se forelesning på mandag 31.10)

$e = 2.71828\ldots$

$$(e^x)' = e^x$$

$$10 = e^{\ln 10}$$

$$(10^x)' = \left(\left(e^{\ln 10}\right)^x\right)'$$

$$= \left(e^{\ln 10 \cdot x}\right)' = e^{\ln 10 \cdot x} \cdot \underbrace{(\ln 10 \cdot x)'}_{\ln 10}$$

så $(10^x)' = \ln 10 \cdot 10^x$

$= 2.30258\ldots \cdot 10^x$

$a = e^{\ln a}$ $\qquad a^x = e^{\ln a \cdot x}$

$$\underline{(a^x)' = (\ln a) \cdot a^x}$$

$x = e^{\ln x}$ deriverer $1 = \underbrace{e^{\ln(x)}}_{x} \cdot (\ln x)'$

$(\ln x)' = \frac{1}{x}$

$\log_a(x) = \frac{\ln x}{\ln a}$

$(\log_a(x))' = \frac{1}{\ln a}(\ln x)' = \underline{\frac{1}{(\ln a)\cdot x}}$

$(\log x)' = \frac{1}{\ln(10)\cdot x}$

$\sim \frac{1}{2.302\ldots} \cdot \frac{1}{x}$

$\sim \underline{0.434\ldots} \cdot \frac{1}{x}$

oppg. Deriver $\frac{\log x}{x^2}$

$(\log x \cdot x^{-2})'$

$= (\log x)' \cdot x^{-2} + (\log x) \cdot (x^{-2})'$

$= \frac{1}{(\ln 10)\cdot x} \cdot \frac{1}{x^2} + (\log x) \cdot (-2x^{-3}) = \underline{\frac{1}{x^3}\left(\frac{1}{\ln 10} - 2\log(x)\right)}$

$\ln x - \ln(x^2 - x) = 2$ løs likningen.

$\ln x + \ln((x^2 - x)^{-1}) = 2$

$\ln\left(\frac{x}{x^2 - x}\right) = 2$

$x \neq 0$

$\ln\left(\frac{x}{x(x-1)}\right) = 2$

$\ln\left(\frac{1}{x-1}\right) = 2$

$-\ln(x-1) = \ln\left(\frac{1}{x-1}\right) = 2$

$\ln(x-1) = (-2)$

$e \qquad e$

$x - 1 = e^{-2}$

$x = 1 + e^{-2} = \underline{1 + \frac{1}{e^2}}$

Deriver $h(x) = \log_2(\sqrt[3]{x})$ | $\log_2 u = \frac{\ln u}{\ln(2)}$.

$= \log_2(x^{1/3}) = \frac{1}{3}\log_2(x)$

Så $(h(x))' = \frac{1}{3}(\log_2 x)' = \frac{1}{3} \cdot \frac{1}{\ln 2} \cdot \frac{1}{x}$

$(\log_2 \sqrt[3]{x})' = \underline{\frac{1}{3\ln(2) \cdot x}}$

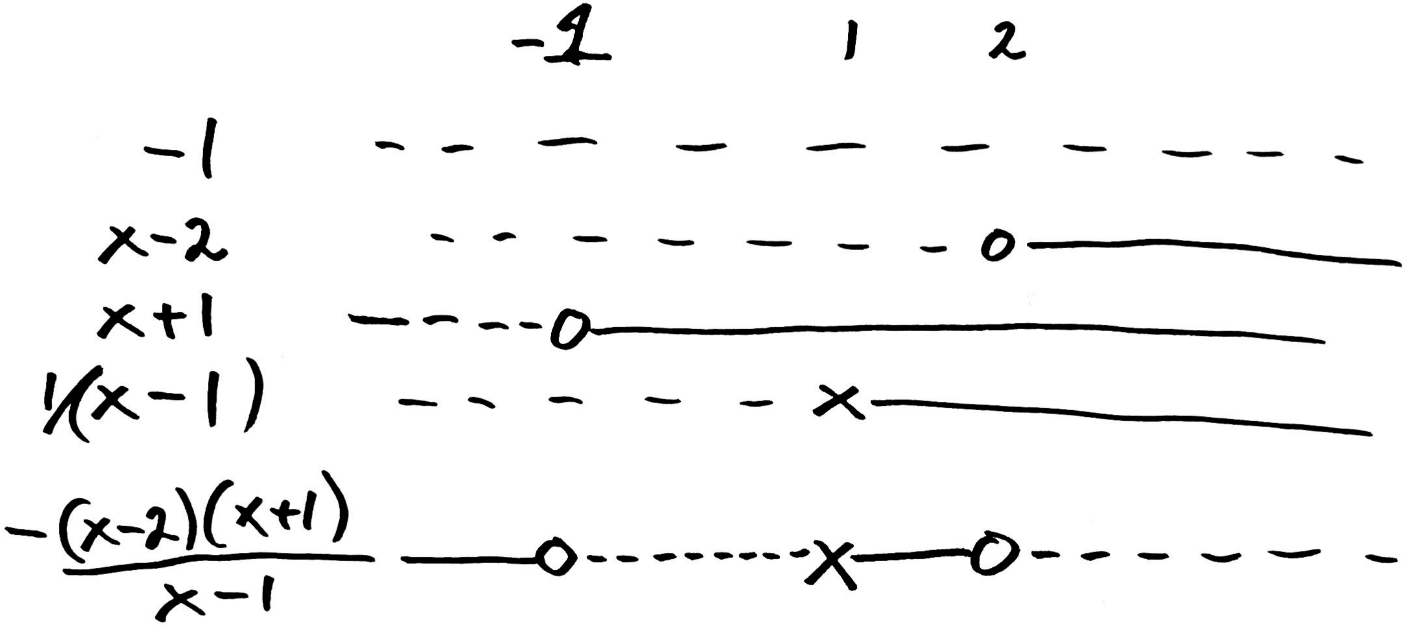
Løs ulikheten $\frac{2}{x-1} \geq x \iff \frac{2}{x-1} - x \geq 0$

$\frac{2}{x-1} - \frac{x(x-1)}{(x-1)} \geq 0 \iff \frac{2 - x(x-1)}{x-1} \geq 0$

$\iff \frac{-x^2 + x + 2}{x-1} \geq 0 \iff \frac{-(x^2 - x - 2)}{x-1} \geq 0$

$\iff \frac{-(x-2)(x+1)}{x-1} \geq 0$

$\iff$



Løsningene er $\underline{x \in \langle -\infty, -1] \cup \langle 1, 2]}$