2.11
2022

# Eksponent funksjon

$a^x$ grunntall $a > 0$
$a \neq 1$

$a^{\log_a x} = x$ logaritme med grunntall $a$.

$\log = \log_{10}$

Logaritmeregler $\log(x \cdot y) = \log x + \log y$

$\log(x^r) = r \cdot \log(x)$

$$\log(2 \cdot 5) = \log 2 + \log 5$$
$$= \log(10^1) = 1$$

$$\log 5 = 1 - \log 2 \quad (\sim 1 - 0.3 = 0.7)$$

$$2^{10} = 1024 \sim 1000 = 10^3$$

$$\log 2^{10} \overset{>}{\sim} \log(10^3)$$

$$10 \log 2 \sim 3$$

så $\log 2 \sim \frac{3}{10} = \underline{0.3}$

Likninger

$$2^x = 5$$

$$\log(2^x) = \log 5$$

$$x \log 2 = \log 5$$

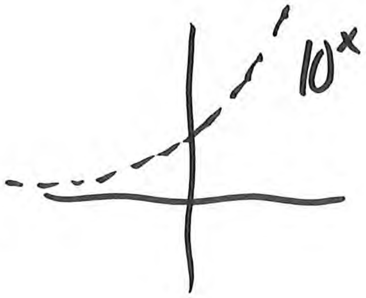
$$x = \frac{\log 5}{\log 2} \sim \frac{0.7}{0.3} = \frac{7}{3} \sim 2.3$$

$$\log x = 2.5$$

$$x = 10^{\log x} = 10^{2.5}$$

$$= 10^{2+\frac{1}{2}} = 10^2 \cdot 10^{1/2}$$

$$x = \underline{100 \cdot \sqrt{10}} \sim \underline{316}$$



$$\log x - \log(1-x) = \frac{1}{3}$$

$$\log x + \log(1-x)^{-1} = \frac{1}{3}$$

$$\log(x \cdot (1-x)^{-1}) = \log\left(\frac{x}{1-x}\right) = \frac{1}{3}$$

$$\Leftrightarrow \frac{x}{1-x} = 10^{\log\left(\frac{x}{1-x}\right)} = 10^{1/3} = \sqrt[3]{10}$$

$$\Leftrightarrow x = (1-x)\sqrt[3]{10} = \sqrt[3]{10} - \sqrt[3]{10} \cdot x$$

$x(1+\sqrt[3]{10}) = \sqrt[3]{10}$ så $\underline{x = \frac{\sqrt[3]{10}}{1+\sqrt[3]{10}}}$

$$\log\left(\frac{1}{a}\right) = \log a^{-1} = -1 \cdot \log a$$

$$\log\left(\frac{1}{a}\right) = -\log a$$

$$\log\left(\frac{b}{a}\right) = \log\left(b \cdot \frac{1}{a}\right) = \log b + \log\left(\frac{1}{a}\right)$$

$$= \log(b) - \log(a).$$

# Test Forkurs Matematikk OsloMet
2. november 2022

**Oppgave 1.** Løs likningen

$$3^x = 5 \cdot 10^x$$

**Oppgave 2.** Løs likningen

$$\log|x+1| = -1$$

**Oppgave 3.** Løs likningen

$$e^{2x} - 2e^x = 24$$

**Oppgave 4.** Følgende uttrykk er lik et rasjonalt tall. Finn dette tallet (uten kalkulator). Benytt gjerne logaritmereglene.

$$3\log(\sqrt{2}) + \log\left(\frac{5}{2}\right) - \frac{1}{2}\log(5)$$

# Test LF

1 $$3^x = 5 \cdot 10^x$$

$$\log 3^x = \log(5 \cdot 10^x)$$

$$x \log 3 = \log 5 + \log(10^x)$$

$$= \log 5 + x$$

$$x \log 3 - x = \log 5$$

$$x(\log 3 - 1) = \log 5$$

$$x = \frac{\log 5}{\log 3 - 1} \sim \underline{-1.337\ldots}$$

2 $$\log |x+1| = -1$$

$$|x+1| = 10^{\log |x+1|} = 10^{-1} = \frac{1}{10}$$

$x+1 = \frac{1}{10}$ : $x = \frac{1}{10} - 1 = \underline{\frac{-9}{10}}$

$x+1 = \frac{-1}{10}$ : $x = \frac{-1}{10} - 1 = \underline{\frac{-11}{10}}$

3 $$e^{2x} - 2e^{x} = 24$$

$$e^{2x} = (e^{x})^{2}$$

$$(e^{x})^{2} - 2e^{x} - 24 = 0$$

$$e^{x} = u$$

$$u^{2} - 2u - 24 = 0$$

$$(u-6)(u+4) = 0$$

$u = 6$ og $u = -4$

$e^{x} = u$ gir $\underbrace{\ln e^{x}}_{x} = \ln u$

$\underline{x = \ln 6}$

($e^{x} = -4$ ingen løsning)

e Euler tallet 2.71828....

$$(e^{x})' = e^{x}$$

$\ln = \log_e$ naturlig logaritme.

$$e^{\ln x} = x$$

4 $3\log(\sqrt{2}) + \log\left(\frac{5}{2}\right) - \frac{1}{2}\log(5)$

$$3 \cdot \log\left(2^{1/2}\right) + \log\left(5 \cdot 2^{-1}\right) - \frac{1}{2}\log 5$$

$$= 3 \cdot \frac{1}{2}\log(2) + \log 5 + \underbrace{\log\left(2^{-1}\right)}_{-\log 2} - \frac{1}{2}\log 5$$

$$= \left(\frac{3}{2} - 1\right)\log(2)$$

$$+ \left(1 - \frac{1}{2}\right)\log 5$$

$$= \frac{1}{2}\log 2 + \frac{1}{2}\log 5$$

$$= \frac{1}{2}(\underbrace{\log 2 + \log 5}_{\log(2 \cdot 5) = \log(10) = 1}) = \underline{\underline{\frac{1}{2}}}$$

# Logaritmisk skala

$\log\left(\frac{E_2}{E_1}\right)$ bel     $E$ effekt.

(desi $= \frac{1}{10}$)     desibel $= \frac{1}{10}$ bel

$\underline{10 \cdot \log\left(\frac{E_2}{E_1}\right)}$ desibel

Lydtrykk $P$

$E$ proporsjonal til $P^2$

$E = k \cdot P^2$

Referanselydtrykk     $P_0 = 20\,\mu\mathrm{Pa}$

( $\mathrm{Pa} = N/m^2$     $\mu = 10^{-6}$ )

grensen av hva som er hørbart for mennesker.

$$10 \cdot \log\left(\frac{kP^2}{kP_0^2}\right) = 10 \log\left(\frac{P^2}{P_0^2}\right) = 10 \log\left(\frac{P}{P_0}\right)^2.$$

$$\underline{= 20 \log\left(\frac{P}{P_0}\right)}$$

$p = p_0$ lydstyrken $0$

dobling av lydtrykk

differansen i lydstyrke $$20 \underbrace{\log\left(\frac{2P}{P_0}\right)}_{\log 2 + \log\left(\frac{P}{P_0}\right)} - 20 \log\left(\frac{P}{P_0}\right)$$

$$= 20 \cdot \log 2 \sim \underline{6\,dB}$$

hørselen : $0\,dB$ til $120\,dB$

$$120 \sim 20 \cdot 6$$

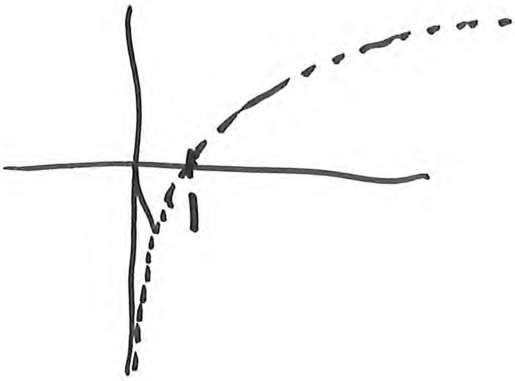
Lydtrykket ved $120\,dB \sim 2^{20} \cdot P_0$

$$= (2^{10})^2 P_0 = (1024)^2 P_0$$

$$= \underline{10^6 \cdot P_0}$$

Deriverte til ln x

$$(\ln x)' = \frac{1}{x}$$



$$(\ln(2x+1))'$$

$$= \frac{1}{2x+1}\,(2x+1)' = \frac{2}{2x+1}$$

$$\ln(3x) = \overset{\text{konst.}}{\ln 3} + \ln x$$

$$(\ln(3x))' = (\ln x)' = \frac{1}{x}$$

Via kjerneregelen

$$(\ln(3x))' = \frac{1}{3x}\cdot(3x)'$$

$$= \frac{3}{3x} = \frac{1}{x}.$$

$$\ln(x^5) = 5\ln x$$

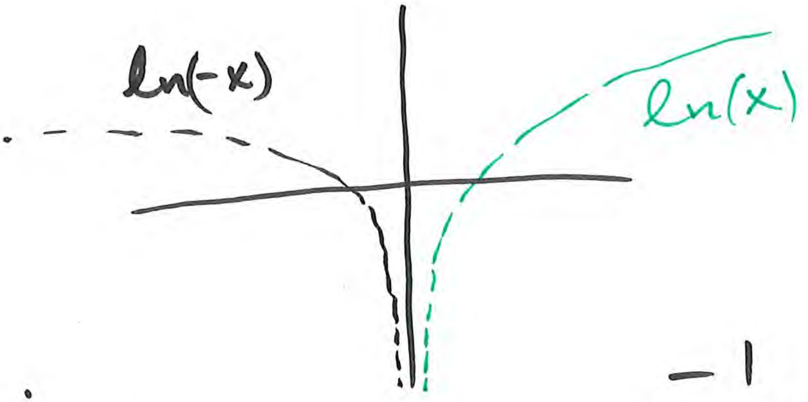
$$(\ln x^5)' = 5(\ln x)' = \frac{5}{x}$$

Via kjerneregelen:

$$(\ln(x^5))' = \frac{1}{x^5}\cdot(x^5)'$$

$$= \frac{5x^4}{x^5} = \underline{\frac{5}{x}}$$

ln(-x)



Kjerneregelen :

$$(\ln(-x))' = \frac{1}{-x} \cdot \overbrace{(-x)'}^{-1}$$

$$= \frac{-1}{-x} = \underline{\frac{1}{x}}$$

$$\ln|x| = \begin{cases} \ln(-x) & x<0 \\ \ln(x) & x>0 \end{cases}.$$

$$\underline{(\ln|x|)' = \frac{1}{x} \text{ for alle } x \neq 0}$$

benytter at $(e^u)' = e^u$ ln x er kjernen

$$(e^{\ln x})' = (x)'$$

Kjerne-regelen

$$\underbrace{e^{\ln x}}_{x} \cdot (\ln x)' = 1$$

deler med x

$$\underline{(\ln x)' = \frac{1}{x}}$$

$$(e^x)' = e^x$$

$$a^x = (\underbrace{e^{\ln a}}_{a})^x$$

$$= e^{\ln a \cdot x}$$

$$\underline{(a^x)' = \ln(a)\, a^x}$$