26.10.22 Produktregelen

$$\underbrace{\big(f(x)\cdot g(x)\big)}_{\text{produkt}}{}' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$(x\cdot x)' = \underbrace{(x)'}_{1}\cdot x + x\cdot \underbrace{(x)'}_{1}$$

$$(x^2)' = 2x$$

(Merk: $(x)'(x)' = 1 \neq 2x = (x^2)'$)

$$\frac{x}{x} = x\cdot\left(\frac{1}{x}\right) = \begin{cases} 1 & x\neq 0 \\ \text{ikke det når } x=0 \end{cases}$$

Den deriverte ($1'=0$) via produktregelen

$$(x)'\cdot\frac{1}{x} + x\cdot\left(\frac{1}{x}\right)'$$

$$1\cdot\frac{1}{x} + x\left(\frac{-1}{x^2}\right) = \frac{1}{x} - \frac{x}{x^2}$$

$$= \frac{1}{x} - \frac{1}{x} = 0 \quad (\text{for alle } x\neq 0)$$

$$\left(\left(\frac{1}{x}\right)' = (x^{-1})' = -1\cdot x^{-1-1} = -1\cdot x^{-2} = (-1)\frac{1}{x^2} = \frac{-1}{x^2}\right)$$

$$\left(x\sqrt{x-2}\right)' = (x)'\sqrt{x-2} + x\left((x-2)^{1/2}\right)'$$

$$= 1\cdot\sqrt{x-2} + x\cdot\tfrac{1}{2}(x-2)^{-1/2}\cdot\underbrace{(x-2)'}_{1}$$

$$= \underline{\sqrt{x-2} + \frac{x}{2\sqrt{x-2}}} = \sqrt{x-2}\,\frac{2\sqrt{x-2}}{2\sqrt{x-2}} + \frac{x}{2\sqrt{x-2}}$$

$$= \frac{2(x-2)+x}{2\sqrt{x-2}} = \underline{\frac{3x-4}{2\sqrt{x-2}}}$$

siden $(\sqrt{x-2})^2 = x-2$

<u>oppg.</u> Deriver $(4x)^3(x^2-3)^5$

$$\left((4x)^3(x^2-3)^5\right)' = \left(4^3\cdot x^3(x^2-3)^5\right)'$$

$$= 4^3\Big[\underbrace{(x^3)'}_{3x^2}(x^2-3)^5 + x^3\underbrace{\left((x^2-3)^5\right)'}_{5(x^2-3)^4\cdot\underbrace{(x^2-3)'}_{2x}}\Big]$$

$$=$$

$$= 4^3\Big[3x^2(x^2-3)^5 + 5\cdot 2x\cdot x^3(x^2-3)^4\Big]$$

$$= 4^3x^2(x^2-3)^4\big[3(x^2-3)+10x^2\big]$$

$$= \underline{4^3x^2(x^2-3)^4\big[13x^2-9\big]}$$

Kurvedrøfting $\quad f(x) = \dfrac{x}{x^2+1}$

horisontal asymptote $\quad$ x-aksen.

$$f'(x) = \left(x \cdot \frac{1}{x^2+1}\right)' = (x)' \frac{1}{x^2+1} + x \left(\frac{1}{x^2+1}\right)'$$

$$= \frac{1}{x^2+1} + x \cdot \frac{-1}{(x^2+1)^2} \cdot \underbrace{(x^2+1)'}_{2x}$$

$$= \frac{(x^2+1)}{(x^2+1)^2} + \frac{-2x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) = 0 \quad$ når $\quad x = 1$ og $x = -1$.

$$\begin{array}{lcccc} & -1 & & 1 & \\ f'(x) \quad - - - - & 0 & \text{———} & 0 & - - - - \end{array}$$

toppunkt $\quad (1, f(1)) = \left(1, \frac{1}{2}\right)$

bunnpunkt $\quad (-1, f(-1)) = \left(-1, \frac{-1}{2}\right)$

Konkavitet:

$$f''(x) = \left((1-x^2) \cdot (1+x^2)^{-2}\right)'$$

$$= (1-x^2)'(1+x^2)^{-2} + (1-x^2)\left((1+x^2)^{-2}\right)'$$

$$-2x\,(1+x^2)^{-2} + (1-x^2)\Big(-2(1+x^2)^{-3}\underbrace{(1+x^2)'}_{2x}$$

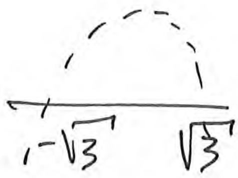
felles nevner =

$$= -2x \frac{1}{(1+x^2)^2} \cdot \frac{1+x^2}{1+x^2} + \frac{-4x(1-x^2)}{(1+x^2)^3}$$

$$= \frac{-2x}{(1+x^2)^3}\left((1+x^2) + 2(1-x^2)\right)$$

$$f''(x) = \frac{-2x}{(1+x^2)^3}\left(3-x^2\right)$$

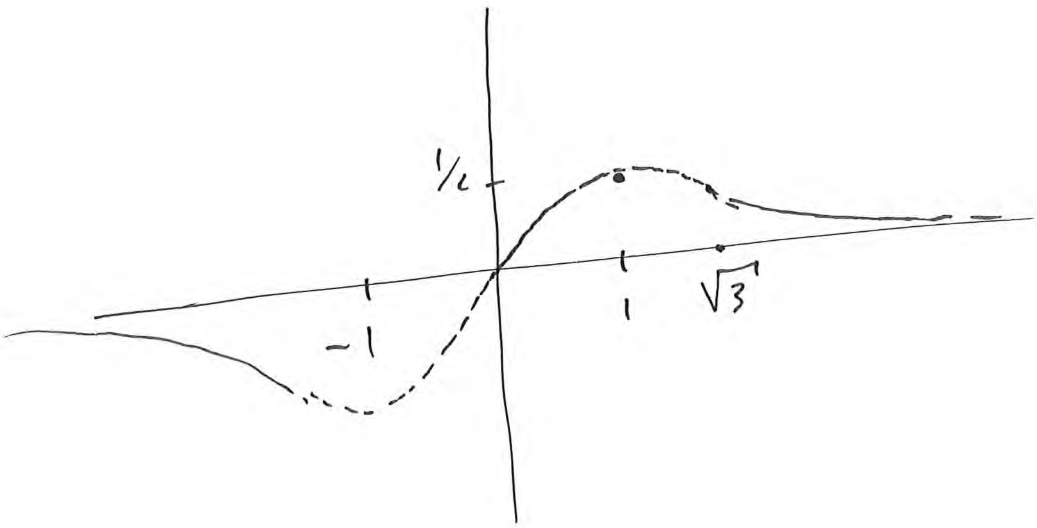
		$-\sqrt{3}$		0		$\sqrt{3}$	
$\frac{1}{(1+x^2)^3}$	+	+	+	+	+	+	+
$-2x$	+	+	+	0	−	−	−
$3-x^2$	−	0	+	+	+	0	−
$f''(x)$	−	0	+	0	−	0	+
Konkavitet	$\cap$		$\cup$		$\cap$		$\cup$



3 vendepunkt

(0.0)

$\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$ og $\left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right)$



Test Forkurs Matematikk OsloMet
26. oktober 2022

Regn uten bruk av hjelpemiddel

Oppgave 1. Deriver funksjonen $(3x-7)^{\sqrt{5}}$

$$\left((3x-7)^{\sqrt{5}}\right)' = \sqrt{5}(3x-7)^{\sqrt{5}-1} \cdot \overbrace{(3x+7)'}^{3}$$

$$= 3\sqrt{5}(3x-7)^{\sqrt{5}-1}$$

Oppgave 2. Deriver funksjonen $\frac{1}{\sqrt[5]{x^3+1}} = \frac{1}{(x^3+1)^{1/5}} =$

$$= \left((x^3+1)^{1/5}\right)^{-1} = (x^3+1)^{-1/5}$$

$$\left((x^3+1)^{-1/5}\right)' = -\frac{1}{5}(x^3+1)^{-\frac{1}{5}-1}\,\overbrace{(x^3+1)'}^{3x^2}$$

Oppgave 3. Deriver funksjonen $x^4(5-x)^5$

$$= \frac{-3x^2}{5} \cdot \frac{1}{\left(\sqrt[5]{1+x^3}\right)^6}$$

Oppgave 4. Gitt funksjonen

$$f(x) = \begin{cases} \frac{x^2+3x+1}{x} & x \neq 0 \\ 3 & x = 0 \end{cases}$$

hvor den vokser og avtar

1) Finn asymptoter 2) bestem monotoniegenskapene 3) finn ekstremalpunkt 4) bestem konkavitet og finn vendepunkt 5) lag en skisse av grafen.

oppg 3 $\quad \left(x^4(5-x)^5\right)'$

$$= (x^4)'\left((5-x)^5\right) + x^4\left((5-x)^5\right)'$$

$$= 4x^3(5-x)^5 + x^4 \cdot 5(5-x)^4 \cdot \underbrace{(5-x)'}_{-1}$$

$$= 4x^3(5-x)^5 - 5x^4(5-x)^4$$

$$= x^3(5-x)^4\left[4(5-x) - 5x\right]$$

$$= x^3(5-x)^4(20-9x)$$

→

oppg 4

$$f(x) = \begin{cases} \dfrac{x^2+3x+1}{x} & x \neq 0 \\ 3 & x = 0 \end{cases}$$

Polynomdivisjon gir:

$$f(x) = \begin{cases} x+3+\dfrac{1}{x} & x \neq 0 \\ 3 & x = 0 \end{cases}.$$

1) $y = x+3$ er en skrå asymptote

fordi: $f(x) - (x+3) = \frac{1}{x}$. $(x \neq 0)$

Denne differensen går mot 0 når

$x \to \infty$ og når $x \longrightarrow -\infty$.

$x = 0$ er en vertikal asymptote fordi

$\lim\limits_{x \to 0^+} f(x) = \infty$ og $\lim\limits_{x \to 0^-} f(x) = -\infty$.

2) $f'(x) = \begin{cases} 1 - \frac{1}{x^2} & x \neq 0 \\ \end{cases}$ ikke kont og da ikke deriverba i $x = 0$.

$f'(x) = \dfrac{x^2-1}{x^2}$ for $x \neq 0$.

$f'(x) > 0$ for $x < -1$ og for $x > 1$.

< 0 for $-1 < x < 1$.

$f(x)$ er stigende i $\langle -\infty, -1]$ og i $[1, \infty\rangle$

— avtagende — $[-1, 0\rangle$ og i $\langle 0, 1]$.

Dette gir:

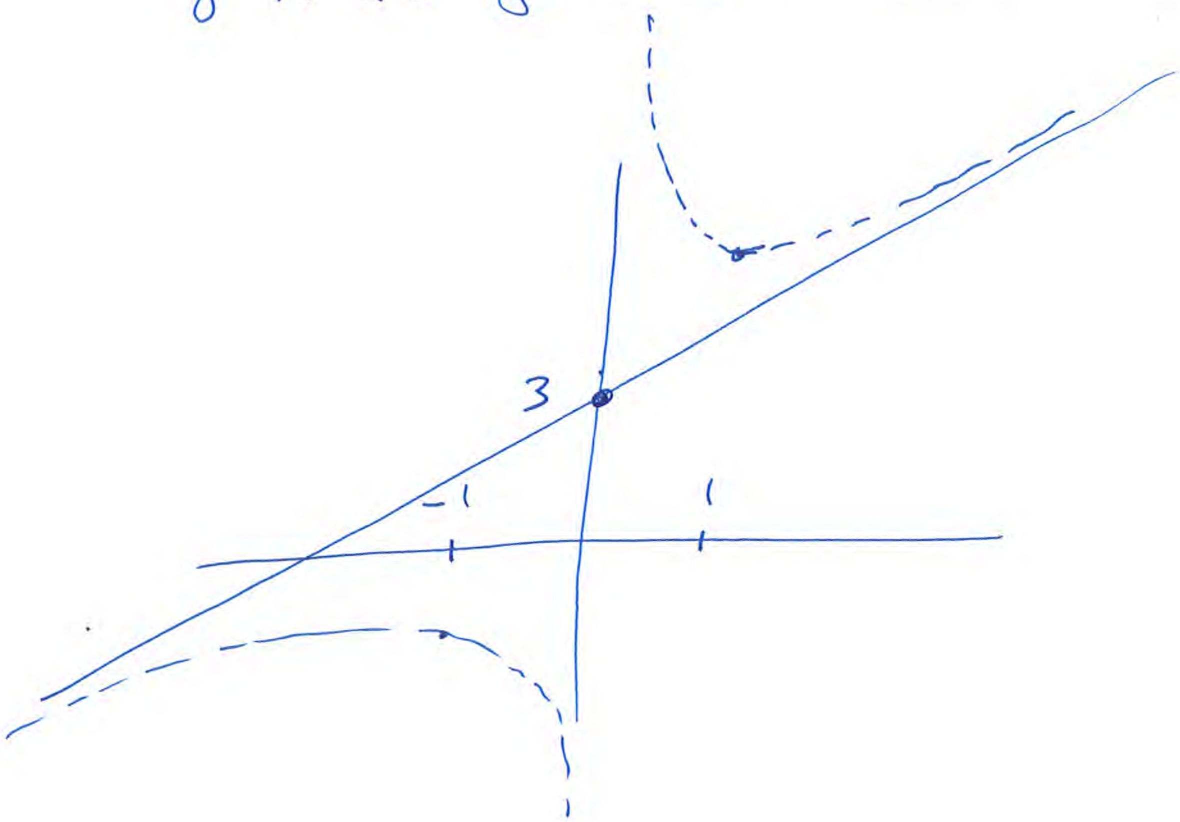
3) toppunkt i $(-1, f(-1)) = (-1, 1)$

bunnpunkt i $(1, f(1)) = (1, 5)$.

4) $f''(x) = \left\{ \frac{2}{x^3} \quad x \neq 0 \right.$

$f''(x) > 0$ for $x > 0$: konkav opp

$f''(x) < 0$ for $x < 0$: konkav ned.



$(0,3)$ er ikkje et vendepunkt :

f skifter konkavitet, men er ikkje kontinuerlig

Kvotientregelen.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g^2(x)}$$

bevis:

$$\left(f(x)\cdot\frac{1}{g(x)}\right)' = f'\cdot\frac{1}{g} + f\,\underbrace{\left(\frac{1}{g}\right)'}_{\frac{-1}{g^2}\cdot g'}$$

$$= \frac{f'}{g}\cdot\frac{g}{g}$$

$$= \frac{f'\cdot g - f\cdot g'}{g^2}$$

Eks

$$\left(\frac{\sqrt{2x-3}}{x^7}\right)' = \frac{\left(\sqrt{2x-3}\right)'\cdot x^7 - \sqrt{2x-3}\,(x^7)'}{(x^7)^2}$$

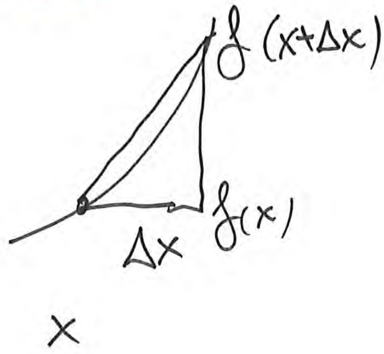
$$= \frac{\frac{1}{\cancel{2}}(2x-3)^{-1/2}\overbrace{(2x-3)'}^{2}\cdot x^7 - \sqrt{2x-3}\cdot 7x^6}{x^{14}}$$

$$= \frac{\frac{1}{\sqrt{2x-3}}\cdot x^7 - \sqrt{2x-3}\cdot 7x^6}{x^{14}}$$

$$= \frac{\frac{x}{\sqrt{2x-3}} - 7\sqrt{2x-3}}{x^8}.$$

Alternativt: $\sqrt{2x-3}\cdot x^{-7}$ og produktregelen...

Vi beviser produktregelen:



$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$\Delta f = f(x+\Delta x) - f(x)$$

$$f(x+\Delta x) = f(x) + \Delta f$$

$$(f \cdot g)' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)\, g(x+\Delta x) - f(x)\, g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(f(x) + \Delta f)(g(x) + \Delta g) - f(x)\, g(x)}{\Delta x}$$

$-f(x)g(x)$

$$= \lim_{\Delta x \to 0} \frac{f(x) \cdot g(x) + f(x) \cdot \Delta g + \Delta f \cdot g(x) + \Delta f\, \Delta g}{\Delta x}$$

$$= \lim_{\Delta x \to 0} f(x) \frac{\Delta g}{\Delta x} + \frac{\Delta f}{\Delta x} \cdot g(x) + \frac{\Delta f}{\Delta x} \cdot \Delta g$$

$$= \underline{f(x)\, g'(x) + f'(x)\, g(x)} \quad (+\,0)$$