24.10.22 7.5-6 Kjerneregelen.

Sammensatt funksjon $f(u(x)) = (f \circ u)(x)$

ytre funksjonen

kjernen

$u(x) = 4 + 3x^2$

$f(u) = u^5$

$(f(x) = x^5, \; f(a) = a^5 \ldots$

$f(u(x)) = (\underbrace{4+3x^2}_{\text{kjerne}})^5$

Kjerneregelen

$$\boxed{(f(u(x)))' = f'(u(x)) \cdot u'(x)}$$

$$u'(x) = (4+3x^2)' = (4)' + 3(x^2)' = 3 \cdot 2x = 6x$$

$$f'(x) = (x^5)' = 5x^4$$

$$(f \circ u(x))' = ((4+3x^2)^5)' = f'(u(x)) \cdot u'(x)$$

$$= 5(4+3x^2)^4 \cdot 6x = \underline{30x(4+3x^2)^4}$$

Mindre detaljert: $((4+3x^2)^5)' = 5(4+3x^2)^4 \cdot (4+3x^2)'$

$$= 5(6x) \cdot (4+3x^2)^4 = \underline{30x(4+3x^2)^4}$$

oppg. Deriver $g(x) = \frac{1}{1+x^4} = (1+x^4)^{-1}$

$$g'(x) = -1(1+x^4)^{-2} \cdot (1+x^4)' = \underline{-\frac{4x^3}{(1+x^4)^2}}$$

Lineær kjerneregel er kjerneregelen hvor kjernen er en lineær funksjon $y = ax + b$

$$\left(f(ax+b)\right)' = f'(ax+b) \cdot \underbrace{(ax+b)'}_{a}$$

$$= \underline{a f'(ax+b)}$$

Sammensetning av to potensfunksjoner

$$(x^5)^r = x^{5 \cdot r}$$

$$(x^2)^3 = x^6$$

Regner ut $\left((x^2)^3\right)'$ med kjerneregelen

$$3(x^2)^2 \cdot (x^2)' = 3x^4 \cdot 2x = 6x^5$$

$$= (x^6)'$$

✓

$$\left((x^s)^r\right)' = r\,(x^s)^{r-1}\cdot(x^s)'$$
$$= r\,x^{s(r-1)}\cdot s\cdot x^{s-1}$$
$$= r\cdot s\,x^{s\cdot r-s+s-1}$$
$$= r\cdot s\,x^{s\cdot r-1} = (x^{sr})'$$

—

$$\left((4+\sqrt{x})^3\right)' = 3(4+\sqrt{x})^2\cdot(4+\sqrt{x})'$$
$$= 3(4+\sqrt{x})^2\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}$$
$$= \underline{\frac{3}{2}(4+\sqrt{x})^2\frac{1}{\sqrt{x}}}.$$

—

$$h(x) = (1+x^3)^2 = 1+2x^3+x^6$$ (kvadratsetningen)

Derivert "direkte":
$$h'(x) = (1+2x^3+x^6)' = 0+2\cdot 3x^2 + 6\cdot x^5 = \underline{6(x^2+x^5)}$$

Derivert av h(x) ved bruk av kjerneregelen.

$$h'(x) = \left(\left(1+x^3\right)^2\right)' = 2(1+x^3)\cdot(1+x^3)'$$
$$= 2(1+x^3)\cdot 3x^2$$
$$= 6x^2(1+x^3) = 6(x^2+\overbrace{x^5}^{x^2\cdot x^3})$$ ✓

$$j(x) = \sqrt{5-x} + (1+x^3)^9$$
$$= (\underbrace{5-x}_{\text{kjerne}})^{1/2} + (\underbrace{1+x^3}_{\text{kjerne}})^9$$

$$j'(x) = \frac{1}{2}(5-x)^{-1/2}\cdot(\underbrace{5-x}_{-1})' + 9(1+x^3)^8\,\underbrace{(1+x^3)'}_{3x^2}$$
$$= \frac{-1}{2\sqrt{5-x}} + 27x^2(1+x^3)^8$$

Oppg Deriver $\sqrt{2+\sqrt{x}} = (2+x^{1/2})^{1/2}$

$$\left(\sqrt{2+\sqrt{x}}\right)' = \frac{1}{2\sqrt{2+\sqrt{x}}} \cdot \left(2+\sqrt{x}\right)'$$

$$= \frac{1}{2\sqrt{2+\sqrt{x}} \cdot 2\sqrt{x}} = \underline{\frac{1}{4\sqrt{x}\sqrt{2+\sqrt{x}}}}$$

En funksjon sammensatt av tre funksjoner

$$f(x) = \sqrt{5+\sqrt{2+\sqrt{x}}}$$

$$f'(x) = \frac{1}{2\sqrt{5+\sqrt{2+\sqrt{x}}}} \cdot \underbrace{\left(5+\sqrt{2+\sqrt{x}}\right)'}_{\frac{1}{4\sqrt{x}\sqrt{2+\sqrt{x}}}}$$

$$= \underline{\frac{1}{8\sqrt{x}\sqrt{2+\sqrt{x}}\sqrt{5+\sqrt{2+\sqrt{x}}}}}$$

Leibniz notasjon for derivasjon

$$f'(x) = \frac{df}{dx}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$\big(f(u(x))\big)' = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta u} \cdot \frac{\Delta u}{\Delta x}$$

$$= f'(u(x)) \cdot u'(x)$$

$\Delta u =$
$u(x+\Delta x) - u(x)$
$\neq 0$ alle
$\Delta x \neq 0$ nær 0.

$$\frac{d(f \cdot u)}{dx} = \frac{df}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$$

$$f(u(x)) = \sqrt[3]{\underbrace{3x + \frac{2}{x}}_{u(x)}}$$

ytre funksjon $f(x) = x^{1/3}$

$= \sqrt[3]{x}$

$$\frac{d\, f \circ u(x)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{3}(u(x))^{\frac{1}{3}-1} \cdot \left(3x + \frac{2}{x}\right)'$$

$$= \frac{1}{3} \cdot \frac{1}{(u(x))^{2/3}} \cdot \left(3 + 2(-x^{-2})\right)$$

$$= \frac{3 - 2/x^2}{3 \cdot \sqrt[3]{(3x + 2/x)^2}}$$

7.5

$$(x^r)' = r x^{r-1}$$

$$r \in \mathbb{R}$$

$$(x > 0)$$

$$\sqrt[n]{x} = x^{1/n}$$

$$\left(\sqrt[n]{x}\right)' = \frac{1}{n} x^{\frac{1}{n}-1}$$

$$= \frac{\sqrt[n]{x}}{n \cdot x}$$

$$= \frac{1}{n\, x^{1-1/n}} = \frac{1}{n\, x^{\frac{n-1}{n}}} = \frac{1}{n \sqrt[n]{(x^{n-1})}}.$$

$$(x^{\sqrt{2}})' = \sqrt{2}\, x^{\sqrt{2}-1}$$

$$(x^{\pi})' = \pi x^{\pi-1}.$$

$$(x^{5.37})' = 5.37 \cdot x^{4.37}.$$

$\sqrt[n]{x} = y$

Vi skal forklare hvorfor

$$\frac{dy}{dx} = \frac{1}{n} x^{\frac{1}{n}-1}.$$

$$\sqrt[n]{x} = y \Rightarrow \left(\sqrt[n]{x}\right)^n = x = y^n$$

så $1 = \frac{d}{dx} x = \frac{d}{dx} y^n = \frac{dy^n}{dy} \cdot \frac{dy}{dx}$

$$1 = n y^{n-1} \cdot \frac{dy}{dx}$$

Derfor er $\frac{dy}{dx} = \frac{1}{n y^{n-1}} = \frac{1}{n\left(\sqrt[n]{x}\right)^{n-1}}$

$$= \frac{1}{n x \cdot \left(\sqrt[n]{x}\right)^{-1}} = \frac{\sqrt[n]{x}}{n x} \checkmark$$

$$= \frac{1}{n} x^{\frac{1}{n}-1}$$

Vi har vist at $(x^{1/n})' = \frac{1}{n} x^{\frac{1}{n}-1}$

for $n \in \mathbb{N}$

Vi bruker kjerneregelen til å vise at $(x^r)' = r x^{r-1}$

$r$ rasjonalt tall.

$n \in \mathbb{N} \quad m \in \mathbb{Z}$

$$x^{m/n} = (\sqrt[n]{x})^m \quad (\text{eller } \sqrt[n]{x^m})$$

$$(x^{m/n})' = \left((\sqrt[n]{x})^m\right)' = m(\sqrt[n]{x})^{m-1} \cdot (\sqrt[n]{x})'$$

$$= m(\sqrt[n]{x})^{m-1} \cdot \frac{1}{n} \frac{\sqrt[n]{x}}{x}$$

$$= \frac{m}{n} (\sqrt[n]{x})^m \cdot \frac{1}{x} = \frac{m}{n} x^{m/n} \cdot \frac{1}{x}$$

$$= \underline{\frac{m}{n} x^{\frac{m}{n}-1}}$$

(som vist tidligere)

Deriver: $\sqrt{x}\cdot\sqrt[3]{x} = x^{1/2}\cdot x^{1/3} = x^{\frac{1}{2}+\frac{1}{3}} = x^{5/6}$

$$\left(\sqrt{x}\sqrt[3]{x}\right)' = \left(x^{5/6}\right)' = \frac{5}{6}\cdot x^{5/6-1}$$

$$= \frac{5}{6}x^{-1/6} = \underline{\frac{5}{6\sqrt[6]{x}}}$$

oppg. Deriver $\dfrac{5}{4\cdot\sqrt[3]{(2-x)^7}} = \dfrac{5}{4}(2-x)^{-7/3}$

$$\left(\frac{5}{4}(2-x)^{-7/3}\right)' = \frac{5}{4}\left(-\frac{7}{3}\right)(2-x)^{-\frac{7}{3}-1}\cdot\underbrace{(2-x)'}_{-1}$$

$$= \frac{35}{12}(2-x)^{-10/3} = \underline{\frac{35}{12\cdot\sqrt[3]{(2-x)^{10}}}}$$

# 7.7 Produktregelen

$(f \cdot g)(x) = f(x) \cdot g(x)$ produktfunksjonen

↑
prikk
gangetegn

Produktregelen

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$x^2 \cdot x^3 = x^5$

$(x^5)' = 5x^4$

(men $(x^2)' \cdot (x^3)'$
$= 2x \cdot 3x^2 = 6x^3 \neq 5x^4$)

$(x^2)' \cdot x^3 + x^2 \cdot (x^3)'$
$= 2x \cdot x^3 + x^2 \cdot 3x^2 = 2x^4 + 3x^4$
$= (2+3)x^4 = 5x^4$ ✓

Oppg Deriver $\sqrt{x}\cdot x$ ved å bruke prod. regelen

$$(\sqrt{x}\cdot x)' = (\sqrt{x})'\cdot x + \sqrt{x}\cdot(x)'$$
$$= \frac{1}{2\sqrt{x}}\cdot x + \sqrt{x}\cdot 1$$
$$= \frac{1}{2}\sqrt{x} + \sqrt{x} = \left(\frac{1}{2}+1\right)\sqrt{x}$$
$$= \underline{\frac{3}{2}\sqrt{x}} = \frac{3}{2}x^{1/2}$$

$$\left(\frac{x}{\sqrt{x}} = \frac{\sqrt{x}\cdot\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x}}\cdot\sqrt{x} = 1\cdot\sqrt{x} = \underline{\sqrt{x}}\right)$$

$$\left(\begin{aligned}\sqrt{x}\cdot x &= x^{1/2}\cdot x = x^{\frac{1}{2}+1} = x^{3/2}\\ (\sqrt{x}\cdot x)' &= (x^{3/2})' = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}\sqrt{x}\end{aligned}\right)$$

Eks $f(x) = (4-3x^2)^7(2+5x)^{11}$

$$f'(x) = ((4-3x^2)^7)'((2+5x)^{11}) + (4-3x^2)^7((2+5x)^{11})'$$

$$f'(x) = 7(4-3x^2)^6 \underbrace{(4-3x^2)'}_{-6x}(2+5x)^{11}$$

$$+(4-3x^2)^7 \cdot 11(2+5x)^{10} \overbrace{(2+5x)'}^{5}$$

$$= -42x(4-3x^2)^6(2+5x)^{11} + 55(4-3x^2)^7(2+5x)^{10}$$

$$= (4-3x^2)^6(2+5x)^{10}\left[-42x(2+5x) + 55(4-3x^2)\right]$$

$$= (4-3x^2)^6(2+5x)^{10}\left[-84x - 210x^2 + 220 - 165x^2\right]$$

$$= (4-3x^2)^6(2+5x)^{10}\left[-375x^2 - 84x + 220\right]$$

=

oppg

Deriver $\frac{4x}{x^2+1} = 4 \cdot x \cdot (x^2+1)^{-1}$

$$\left(\frac{4x}{x^2+1}\right)' = 4\left(\underbrace{(x)'}_{1}(x^2+1)^{-1} + x\underbrace{\left((x^2+1)^{-1}\right)'}_{-(x^2+1)^{-2}\underbrace{(x^2+1)'}_{2x}}\right)$$

$$= 4\left(\frac{1}{x^2+1} - \frac{2x \cdot x}{(x^2+1)^2}\right)$$

$$= 4\left(\frac{(x^2+1) - 2x^2}{(x^2+1)^2}\right) = \underline{4\,\frac{1-x^2}{(x^2+1)^2}}$$