19.sep. Kap 5 **Polynomer**

(1) x variabel

$x \cdot x \cdot x = x^3$

Ganger og legger sammen x og tall giv **polynomer**

$7x^6 - 5x^4 + 3x + 11$

Generelt polynom

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$\deg(p(x)) = n$ hvis $a_n \neq 0$

$\deg(13) = 0$ $\deg(0) = -1$

$\deg(0 \cdot x^5 + x^3) = 3$

opg:

$$\deg\big(\underbrace{x^3 - x(x+1)(x-1)}_{x^3 - x(x^2-1) = x}\big) = \deg(x) = 1$$

$$\deg\big(2x^7 + x^5 - x^2(2x^5+3)\big) = 5.$$

$2x^7 + x^5 - 2x^7 - 3x^2$

produktet av to polynomer er et polynom.

2) summen ______

Rasjonale funksjoner $\frac{p(x)}{q(x)}$ $p(x)$, $q(x)$

$\frac{1}{q(x)} \cdot q(x) = 1$ bortsett fra x hvor $q(x) = 0$.

$\deg \frac{p(x)}{q(x)} = \deg p(x) - \deg q(x)$.

$\deg \left(\frac{1}{x}\right) = -1$

Hvis $p(a) = 0$ så deler $(x-a)$ polynomet P.

Så polynom P av grad n har n eller færre nullpunkt

$p(x) = 0$

slider for n.

Tegnet polynomer i geogebra, blant annet $x(x-1)\cdots(x-n)$ ved Product(x-i, i, 0, n)

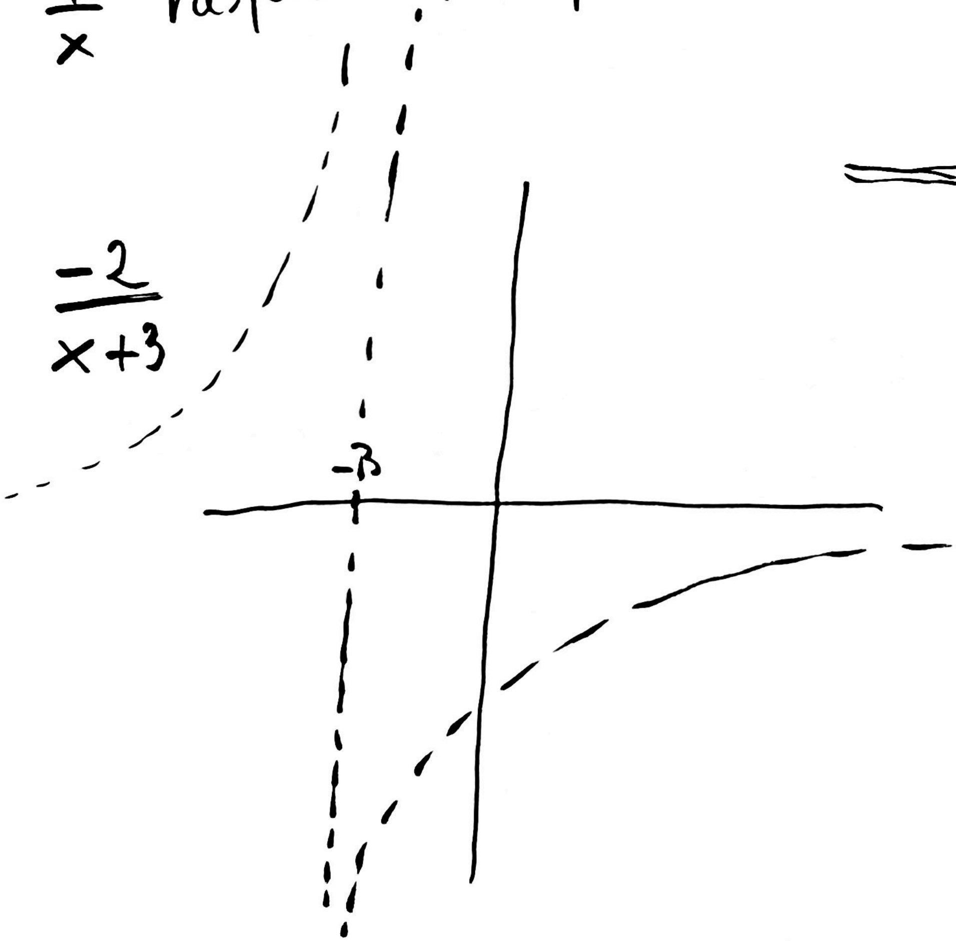
③ n odde $p(x)$ har minst én rot

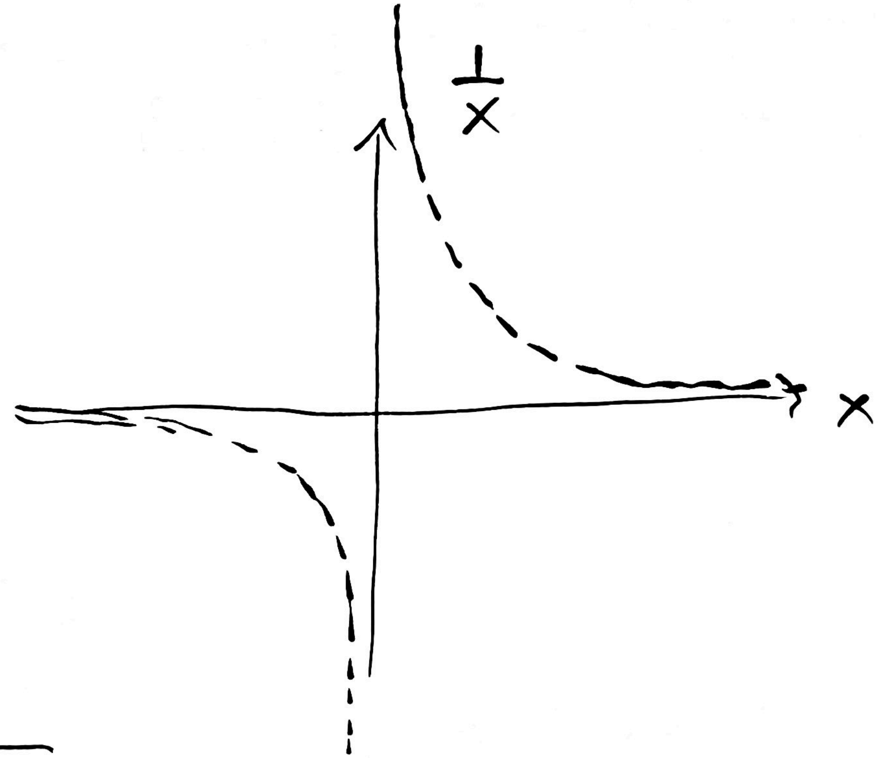
n jevn $p(x)$ behøver ikkje ha røtter:

$$x^{2m}+1 = (x^m)^2+1 \geq 1.$$

$\frac{1}{x}$ rasjonal funksjon

$\frac{-2}{x+3}$





Tegnet $\frac{1}{1+x^2}$ $\qquad$ $\frac{1}{x(x-1)\cdots(x-n)}$ $\qquad$ i geogebra.

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$p(x)$ og $q(x)$ to forskjellige polynomer.

Da er $p(x) = q(x)$ bare for et endelig antall x.

(antallet $\leq \deg(p-q)$)

$$\frac{\text{polynom}}{p(x)-q(x)} = 0$$

$\neq 0$

Polynomer i flere variabler:

x^2+y^2-1 $\qquad$ $xz + 3x^2y - 7z^2$

pol i x, y $\qquad$ pol. i x, y, z

$y - x^2$

5 x^4+1 er et produkt av 2 annengradsfaktorer

$$= \underbrace{(x^2+1)^2}_{x^4+2x^2+1} - 2x^2 = (x^2+1)^2 - (\sqrt{2}x)^2$$

$$x^4+1 = (x^2+\sqrt{2}\cdot x+1)(x^2-\sqrt{2}x+1)$$

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Polynom divisjon

$$\frac{x^3+2x}{x^2+1}, \qquad x^3 = x(x^2+1) - \underset{\text{lavere grad}}{x}$$

$$= \frac{x(x^2+1) - x + 2x}{x^2+1} = \underset{\text{polynom}}{x} + \frac{x}{x^2+1}$$

lavere grad enn x^2+1

$$\frac{p(x)}{q(x)} = \underset{\text{kvotient}}{s(x)} + \underset{\text{rest}}{\frac{r(x)}{q(x)}}$$

$\underline{\deg r < \deg q}$

Alternativ formulering. $p(x) = s(x) \cdot q(x) + r(x)$

(alle x s.a $q(x) \neq 0$, og derfor like for alle x.)

$$p(x) : q(x) = s(x) + \frac{r(x)}{q(x)}$$

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Eks

$$2x^2+3x-1 : x-2 = \underline{2x+7+\frac{13}{x-2}}$$

$$-\left(2x^2-4x\right)+0$$

$$0 \quad 7x-1$$

$$7x-14$$

$$0+13$$

Eks

$$\frac{x^3-2x-1}{2x^2-3}$$

$$x^3 \quad -2x-1 : 2x^2-3 = \underline{\frac{1}{2}x+\frac{-\frac{1}{2}x-1}{2x^2-3}}$$

$$x^3 \quad -\frac{3}{2}x$$

$$0 \quad -\frac{1}{2}x-1$$

Eks

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$$\frac{x^3+1}{3x+1}$$

$$x^3 + \quad + \quad + 1 : 3x+1 = \underline{\tfrac{1}{3}x^2 - \tfrac{1}{9}x + \tfrac{1}{27} + \frac{26/27}{3x+1}}$$

$$\underline{x^3 + \tfrac{1}{3}x^2}$$

$$0 - \tfrac{1}{3}x^2 \quad + 1$$

$$\underline{-\tfrac{1}{3}x^2 - \tfrac{1}{9}x + 0}$$

$$\tfrac{1}{9}x + 1$$

$$\underline{\tfrac{1}{9}x + \tfrac{1}{27}}$$

$$0 \quad \frac{26}{27}$$

"Nevneren" lineær.

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$$\frac{p(x)}{x-a} = S(x) + \frac{r}{x-a}$$

$$p(x) = S(x)(x-a) + r$$ ← deg ≤ 0 konstant.

$x=a$: $p(a) = r$

$$\boxed{(x-a) \text{ deler } p(x) \iff \text{resten, } p(a) = 0}$$

Deler $x-1$ polynomet $p(x) = x^4 + 2x^2 - 3$ ja
$q(x) = x^5 - 3x^2 + 1$ nei.

$p(1) = 1+2-3 = 0$

$q(1) = 1-3+1 = -1$

Faktoriser $p(x) = x^3 + x^2 - 2$.

10 Leter etter små heltallige røtter:

$p(1) = 1^3 + 1^2 - 2 = 0$ så $x=1$ er en rot

så $x-1$ deler $p(x)$.

$$
\begin{array}{l}
x^3 + x^2 \quad - 2 : x - 1 = x^2 + 2x + 2 \\
\underline{x^3 - x^2} \\
\quad 2x^2 \quad - 2 \\
\quad \underline{2x^2 - 2x} \\
\qquad 2x - 2 \\
\qquad \underline{2x - 2} \\
\qquad\quad 0
\end{array}
$$

x^2+2x+2 irredusible siden $(x+1)^2 + 1 \geq 1$

$$x^3 + x^2 - 2 = \underline{(x-1)(x^2+2x+2)}$$

11 Faktoriser $p(x) = x^3 - x^2 - 3x + 3$

$p(1) = 0$ så $x-1$ er en faktor i $p(x)$

$$
\begin{array}{llll}
x^3 & -x^2 & -3x+3 & : x-1 = x^2 - 3 \\
x^3 & -x^2 & & \\
\hline
0 & 0 & -3x+3 & \\
 & & -3x+3 & \\
\hline
 & & 0 &
\end{array}
$$

(konjugatsetningen

$$x^3 - x^2 - 3x + 3 = (x-1)(x^2-3)$$

$$= \underline{(x-1)(x+\sqrt{3})(x-\sqrt{3})}$$

12 Når er $q(x)=x^3+2x^2+cx+3$ delelig med $x+2$.

$(x-(-2))$

Det skjer når $q(-2)=0$.

$$(-2)^3+2(-2)^2+c(-2)+3=0$$
$$-8+8-2c+3=0$$
$$\underline{c=\tfrac{3}{2}}.$$

Forkort om mulig $\dfrac{x^2+3x+2}{x^3+x+1}$ ikke mulig å forkorte.

$x^2+3x+2=(x+2)(x+1)$

$q(x)=x^3+x+1$

$q(-1)=-1$

$q(-2)=-8-2+1=-9$

så $x+1$ og $x+2$ er ikke faktorer i x^3+x+1

opg 13

Forkort $\dfrac{x-1}{x^3+2x-3}$

Er $x-1$ en faktor i x^3+2x-3?

$1^3+2\cdot 1-3=0$. Ja.

$x^3 + 2x - 3 : x-1 = x^2+x+3$

$$\begin{array}{l} x^3 \quad + 2x - 3 \\ \underline{x^3 - x^2} \\ 0 + x^2 + 2x - 3 \\ \underline{x^2 - x} \\ 3x - 3 \\ \underline{3x - 3} \\ 0 \end{array}$$

Så $\dfrac{x-1}{x^3+2x-3}$

$= \dfrac{x-1}{(x-1)(x^2+x+3)} = \dfrac{1}{x^2+x+3}$

(14)

opg $r(x) = x^3 + 2x^2 - 5x - 6$

Faktoriser $r(x)$.

$r(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = -1 + 2 + 5 - 6 = 0$

$(x - (-1)) = x + 1$ er en faktor i $r(x)$.

$(r(2) = 8 + 8 - 10 - 6 = 0)$

$$
\begin{array}{l}
x^3 + 2x^2 - 5x - 6 : x + 1 = x^2 + x - 6 \\
\underline{x^3 + x^2} \qquad\qquad\qquad\qquad\quad = (x-2)(x+3) \\
\quad x^2 - 5x - 6 \\
\quad \underline{x^2 + x} \\
\qquad -6x - 6 \\
\qquad \underline{-6x - 6} \\
\qquad\quad 0
\end{array}
$$

$r(x) = \underline{(x+1)(x-2)(x+3)}$

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$y = ax^2 + bx + c$ parabelen går gjennom (x_1, y_1)

$\Leftrightarrow y_1 = \underline{a} x_1^2 + \underline{b} x_1 + \underline{c}$ likning i a, b, c

Finn parabelen som går gjennom:
$(1, 1)$
$(-1, 2)$
$(2, 4)$

L1 $1 = a + b + c$

L2 $2 = a - b + c$

L3 $4 = 4a + 2b + c$

$y = \frac{7}{6}x^2 - 0.5 \cdot x + \frac{1}{3}$.

Fra L1: $c = 1 - a - b$ setter inn i L2 og L3

$2 = a - b + 1 - a - b = 1 - 2b$ gir $\underline{b = \frac{-1}{2}}$

$4 = 4a + 2b + 1 - a - b = 3a + b + 1$

$3a = 4 - 1 - b = 3 - b = \underline{3 + \frac{1}{2}}$ så $a = \frac{7}{6}$

$c = 1 - a - b = 1 - \frac{\left(3 + \frac{1}{2}\right)}{2} - \left(-\frac{1}{2}\right) = 1 - \frac{7}{6} + \frac{1}{2} = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$

Tegnet opp i geogebra og sjekket.