12 sep 22

Kap 4 Faktorisering

$$(x+3)(x-2) = x(x-2) + 3(x-2) = x^2 - 2x + 3x - 6$$
$$= x^2 + x - 6$$

$\longrightarrow$ ganger ut

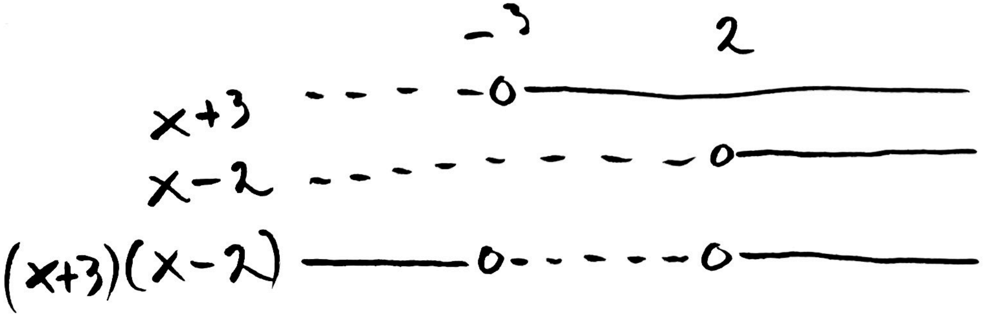
$\longleftarrow$ faktoriserer

$$-3x^2(x-2) = -3x^3 + 6x^2$$

$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0 \iff x+3=0 \text{ eller } x-2=0$$

Løsningene er $\underline{x = -3, 2}$

$$x^2 + x > 6 \iff x^2 + x - 6 > 0 \iff (x+3)(x-2) > 0$$



Løsningsmengden er $\underline{\langle -\infty, -3 \rangle \cup \langle 2, \infty \rangle}$

polynomer summer av $a_i x^i$ $i \geq 0$

koeffisient til x^i $a_0 x^0 = a_0$, $a_1 x^1 = a_1 x$, $a_2 x^2$, $a_3 x^3$, ...

$a_i x^i$ kalles monom.

polynom	3,	$2x+1$,	x^2-7,	$-2x^3+x-4$,	x^5+7, ...
grad	0	1	2	3	5

produkt av polynomer er igjen et polynom

graden ~~ga~~ legges sammen

$$\underset{2}{x^2}\underset{1}{(x+3)}\underset{1}{(x-2)} = \underset{2}{x^2}\underset{2}{(x^2+x-6)} = \underset{4}{x^4+x^3-6x^2}$$

grad 2 1 1

(sum $(x^2+1) + (-x^2+x) = x+1$

grad 2 2 1)

2 grads polynom $p(x) = ax^2 + bx + c$ $a \neq 0$

2 gradslikning $p(x) = 0$ grad $p = 2$

n-te grads likning $p(x) = 0$ grad $p = n$

$x^4 - 16 = 0$ 4. gradslikning.

Løsningene kalles for <u>røtter</u> til polynomet

Rasjonalt uttrykk $R(x) = \frac{p(x)}{q(x)}$ $p(x)$, $q(x)$ polynomer

$\frac{x+1}{x^2-3x}$, $p(x) = \frac{p(x)}{1}$, $\frac{1}{x}$

4.3 "Heltallsmetoden" $x^2 + bx + c$ $\quad b, c \in \mathbb{Z}$

$$(x+p)(x+q) = x^2 + (p+q)x + p \cdot q.$$

Søker p, q slik at $\quad p + q = b$

$p \cdot q = c$

$q(x) = x^2 + 5x + 6$ $\qquad 6 = 2 \cdot 3 = 1 \cdot 6$

$2 + 3 = 5$

$q(x) = (x+2)(x+3)$

$x^2 + 7x + 6$

$= (x+6)(x+1)$

$x^2 + 5x - 6$

$= (x+6)(x-1)$

$p + q = 5 \quad = 6 + (-1)$

$p \cdot q = -6 = (-1) \cdot 6 = 1 \cdot (-6)$

$(-2) \cdot 3 = 2 \cdot (-3$

$$p(x) = x^2 - 2x - 35 = (x+5)(x-7)$$

$p \cdot q = -35 = (-7) \cdot 5$

$p + q = -2 = (-7) \cdot 5$

$p(x) = 0 \Leftrightarrow x+5=0$ eller $x-7=0$

Løsninger $x = \underline{-5, 7}$.

gcg

$$q(x) = x^2 + 4x - 12 = (x+6)(x-2)$$

$-12 = 6(-2)$

$4 = 6 - 2$

$q(x) = 0$

$(x+6)(x-2) = 0$

$x+6 = 0$ eller $x-2=0$

$\underline{x = -6, 2}$

$q(x) > 0$

Løsningsmengde

$\underline{\langle -\infty, -6 \rangle \cup \langle 2, \infty \rangle}$

~~opg~~

$p(x) = x^2 + 11x + 30$ $\qquad 30 = 2 \cdot 3 \cdot 5 = 6 \cdot 5$

$\underline{= (x+5)(x+6)}$ $\qquad 11 = 6 + 5$

$p(x) = 0$

$(x+5)(x+6) = 0 \iff x+5 = 0$ eller $x+6 = 0$

$x = -5 \qquad x = -6$

Løsningene er -5 og -6.

$(x+p)(x+q) = x^2 + (p+q)x + p \cdot q$

$(x+p)(x-p) = x^2 - p^2$ Konjugatsetningen

$p = -q$

$(x+p)(x+p) = (x+p)^2 = x^2 + 2px + p^2$ Kvadratsetningen

$p = q$

$x^2 - 7 = x^2 - (\sqrt{7})^2 = (x+\sqrt{7})(x-\sqrt{7})$

Røtten er $\underline{\pm\sqrt{7}}$

$x^2 = c$ har røtter $x = \pm\sqrt{c}$

$$x^2 - c = 0$$

$$(x + \sqrt{c})(x - \sqrt{c}) = 0$$

opg Faktoriser $4x^2 - 12$ og finn røttene.

$$= 4(x^2 - 3)$$

$$= 4(x^2 - \sqrt{3}^2)$$

$$= \underline{4(x+\sqrt{3})(x-\sqrt{3})}$$

Røttene er $\underline{\pm\sqrt{3}}$

<u>eks</u>

$$4x^4 - 9 = 4\left((x^2)^2 - \frac{9}{4}\right) = 4\left((x^2)^2 - \left(\frac{3}{2}\right)^2\right)$$

$$= 4\left(x^2 + \frac{3}{2}\right)\left(x^2 - \frac{3}{2}\right)$$

$$= 4\left(x^2 + \frac{3}{2}\right)\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)$$

$$4x^2 - 1 = (2x)^2 - 1 = (2x+1)(2x-1)$$

$$4\left(x^2 - \frac{1}{4}\right) = 4\left(x^2 - \left(\frac{1}{2}\right)^2\right) = 4\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$$

$$x^2 + 6x + 9 = (x+3)^2$$

$$x^2 + 2\sqrt{2}\,x + 2 = (x+\sqrt{2})^2$$

$$x^2 + 2px + p^2 = (x+p)^2$$

$$4x^2 + 16x + 16 = 4x^2 + 4 \cdot 4x + 4 \cdot 4$$

$$= 4(x^2 + 4x + 4) = \underline{4(x+2)^2}$$

opg

$$2x^2 - 20x + 50$$

$$= 2(x^2 - 10x + 25)$$

$$= 2(x-5)^2$$

Det er én rot.

Den er $\underline{x = 5}$

Rasjonale uttrykk

$$\frac{x^2+5x+6}{x^2+6x+9} \underset{\text{Faktorisert}}{=} \frac{(x+2)(x+3)}{(x+3)^2}$$

$$= \frac{x+2}{x+3} \cdot \overbrace{\frac{x+3}{x+3}}^{1} = \underline{\frac{x+2}{x+3}}$$

Forkortet

$$* \quad \frac{1}{x} + \frac{4}{x-2} = \frac{1}{x} \cdot \frac{x-2}{x-2} + \frac{4}{x-2} \cdot \frac{x}{x}$$

$$= \frac{(x-2) + 4x}{x(x-2)} = \underline{\frac{5x-2}{x(x-2)}}$$

$$* \quad \frac{x^2+5x+6}{x^2+6x+9} + \frac{x^2}{x^2+3x} = \frac{(x^2+5x+6)(x^2+3x) + x^2(x^2+6x+9)}{(x^2+6x+9)(x^2+3x)} \ \ldots$$

Forkorter først:

$$\frac{x+2}{x+3} + \frac{x}{x} \cdot \frac{x}{x+3} = \frac{(x+2)+x}{x+3} = \frac{2x+2}{x+3} = \underline{\frac{2(x+1)}{x+3}}$$

opg. $\frac{x^2+4x+4}{x^2-4} = \frac{(x+2)^2}{(x+2)(x-2)} = \frac{x+2}{x+2} \cdot \frac{x+2}{x-2}$

$= \frac{x+2}{x-2}$ når $x \neq -2$.

$= (x+2)/(x-2)$ $\left(\neq x+2/x-2 \right)$, $\left(\neq x+2/(x-2) \right)$

Løs likningen $\frac{x^2+4x+4}{x^2-4} = 0 \iff \frac{(x+2)^2}{(x+2)(x-2)} = 0$

$\iff \frac{x+2}{x-2} = 0$ for $x \neq -2$.

<u>ingen røtter</u>.

Løs ulikheten $\frac{1}{x-2} \leq \frac{-1}{4}$.

$\iff \frac{4}{x-2} + 1 \leq 0 \iff \frac{4}{x-2} + \frac{x-2}{x-2} \leq 0 \iff \frac{x+2}{x-2} \leq 0$

	-2	2	
$x+2$	- - - - 0	——————	——————
$1/(x-2)$	- - - - - -	- - - - x	——————
$(x+2)/(x-2)$	—————— 0	- - - - - - x	——————

Løsningene er

$\left(x \in \langle -\infty, -2] \cup \langle 2, \infty \rangle\right)$ ← Løsning til $\frac{1}{x-2} \geq \frac{-1}{4}$

$\underline{}$

$x \in [-2, 2\rangle$

4.5, 6 og 8 i boka

2 gradsformelen

$$ax^2 + bx + c = 0$$

Løsningen er $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$D = b^2 - 4ac$ diskriminanten

$ax^2 + bx + c$ ikkje faktoriseres over $\mathbb{R}$

$D < 0$ ingen reelle løsninger

$D = 0$ én løsning $\frac{-b}{2a}$

$$ax^2 + bx + c$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$
$$= a\left(x + \frac{b}{2a}\right)^2$$

$D > 0$ to løsninger x_1, x_2

$$ax^2 + bx + c$$
$$= a(x - x_1)(x - x_2)$$

$$x^2+1 = 1\cdot x^2 + 0\cdot x + 1$$

$a=1 \quad b=0 \quad c=1$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{\pm\sqrt{-4}}{2}$$ ingen reelle løsringer

$$x^2-6x+9$$

$a=1 \quad b=-6$

$c=9$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4\cdot 1\cdot 9}}{2\cdot 1}$$

$$= \frac{6 \pm \sqrt{36-36}}{2} = \frac{6}{2} = 3$$

én rot: $\underline{x=3}$

$$x^2-6x+9 = (x-3)^2$$

$$x^2 + 2x - 5$$

$a = 1 \quad b = 2 \quad c = -5$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1(-5)}}{2 \cdot 1} = -1 \pm \frac{\sqrt{4 + 4 \cdot 5}}{\sqrt{4}}$$

To røtter

$$= -1 \pm \sqrt{\frac{4(1+5)}{4}} = 1 \pm \sqrt{6}$$

$$x^2 + 2x - 5 = (x - 1 - \sqrt{6})(x - 1 + \sqrt{6})$$

*

$$3x^2 + x - 1$$

$a = 3 \quad b = 1 \quad c = -1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3(-1)}}{2 \cdot 3} = \frac{-1 \pm \sqrt{13}}{6}$$

$$3x^2 + x - 1 = 3 \cdot \left(x + \frac{1 - \sqrt{13}}{6}\right)\left(x + \frac{1 + \sqrt{13}}{6}\right)$$

$x^2 - 3 = 0$

$x^2 + 0 \cdot x - 3$

$a=1 \quad b=0 \quad c=-3$

2.gradsformelen

$$x = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1(-3)}}{2 \cdot 1}$$

$$= \pm \frac{\sqrt{4 \cdot 3}}{2} = \pm \frac{\sqrt{4}}{2} \cdot \sqrt{3}$$

$$= \underline{\pm \sqrt{3}}$$

tungvint

Lettere: $x^2 - 3 = 0 \iff x^2 = 3 \iff x = \pm\sqrt{3}$.

$\left((x - \sqrt{3})(x + \sqrt{3}) = 0 \text{ etc..} \right)$

$(x+2)(x+3) = x^2 + 5x + 6$ røttene er $-3, -2$.

opg

Bruk: 2gradsformelen til å finne røttene:

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2}$$

$= -2.5 \pm \frac{1}{2}$

Løsningene er $\underline{-3, -2}$.

$$x^5 - 3x^3 + x = 0$$

$$x(x^4 - 3x^2 + 1) = 0$$

$$x((x^2)^2 - 3(x^2) + 1) = 0$$

$x = 0$ eller

$$(x^2)^2 - 3x^2 + 1 = 0$$

$$x^2 = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{3 \pm \sqrt{5}}{2}$$

$$\underline{x = \pm\sqrt{\frac{3+\sqrt{5}}{2}}} \quad \text{og} \quad \underline{\pm\sqrt{\frac{3-\sqrt{5}}{2}}}$$

5 løsninger.

$P(x)$ polynom av grad $n \geq 0$

$P(x) = 0$ ha n eller færre løsninger

$P(x)$ faktoriserer som et produkt av første og annengrads polynomer over $\mathbb{R}$

Fullføring av kvadrat

$$\left(x+b/2\right)^2 = x^2+bx+\left(\frac{b}{2}\right)^2$$

$$x^2+bx+c = \left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$$

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\underbrace{\frac{c}{a}-\left(\frac{b}{2a}\right)^2}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4\cdot a^2}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4\cdot a^2}\right) = a\left(\left(x+\frac{b}{2a}\right)^2-\sqrt{\frac{b^2-4ac}{4a^2}}^{\,2}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2-\left(\frac{\sqrt{b^2-4ac}}{2|a|}\right)^2\right) \underset{\text{konj. set.}}{=}$$

$$a\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2|a|}\right)\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2|a|}\right)$$

$$\pm \frac{1}{|a|} = \pm \frac{1}{a} \quad (\text{eventuelt } \mp \frac{1}{a})$$

$$ax^2 + bx + c = a\left(x + \frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\right)$$

Røttene er $\underline{x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}}$