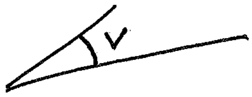
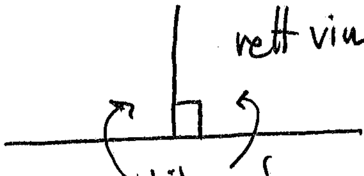
①

6 Trigonometri og geometri



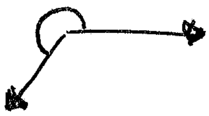


rett vinkel

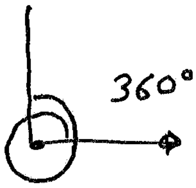
like store vinkler

180°

En rett vinkel er $\frac{180°}{2} = 90°$



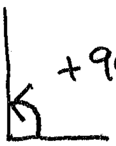
Vinkler kan utvides fra $[0°, 360°]$ (et helt omløp) til alle reelle tall.



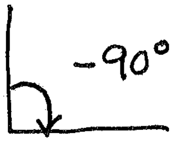
$360° + 90° = 450°$

Velge positiv retning til en vinkel

mot klokken

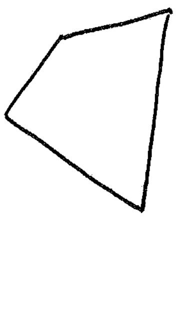


$+90°$

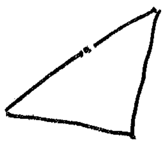


$-90°$

4-kant



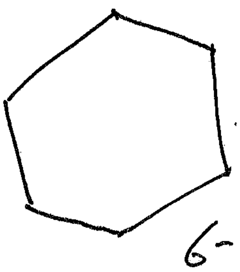
4 rette sider



3-kant

n kant

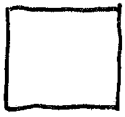
$n \geq 3$



6-kant.

4-kanter :

rektangel
(alle vinklene er rette)

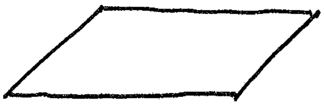


kvadrat

alle sidene i rektangelet er like lange.

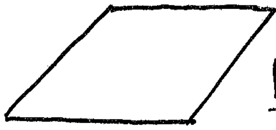
trapes

to sider parallelle



parallellogram

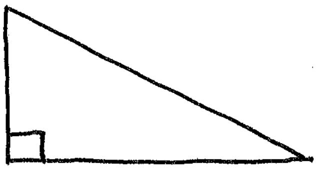
to og to sider er parallelle



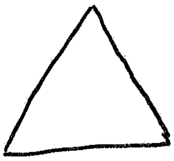
rombe

parallellogram hvor alle sidene er like lange

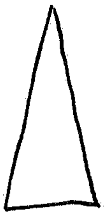
3-kanter



rettvinkla trekant

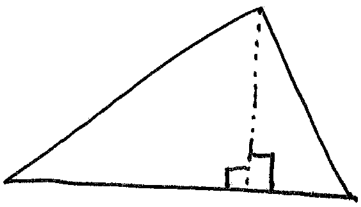


likesida trekanter
(alle tre sidene er like lange)

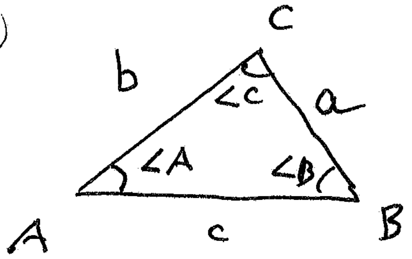


likebeinet trekant
(to av sidene er like lange)

En trekant er satt sammen av to rettvinkla trekanter.



③

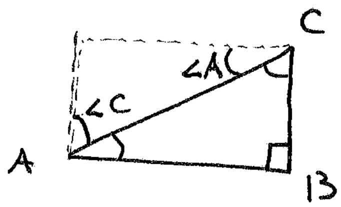


- vinklen i hjørne A : $\angle A$ eller bare A.
- side a . (motsatt hjørne A) vi bruker også a om lengden til siden

Summen av vinklene i en trekant er alltid 180 grader

$$\angle A + \angle B + \angle C = 180°$$

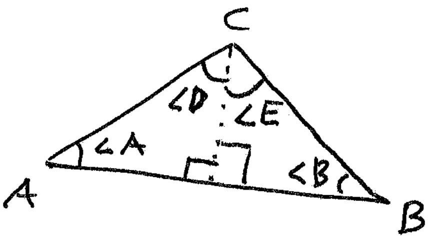
Forklaring:



$\angle A + \angle C = 90°$

$\angle B = 90°$

så $\angle A + \angle B + \angle C = 180°$ i en rettvinklet △



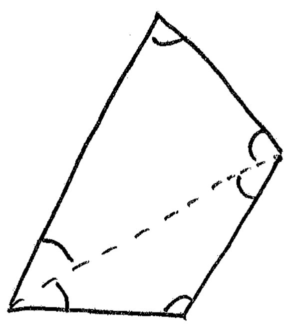
$\angle A + \angle D = 90°$

$\angle B + \angle E = 90°$ (siden rettvinkla trekanter)

Legger sammen $\angle A + \angle B + \underbrace{(\angle D + \angle E)}_{} = 180°$

$$\angle A + \angle B + \angle C = 180°$$

Summen av vinklene i en 4-kant er 360 grader



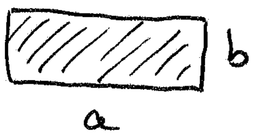
Summen av vinklene i firkanten er summen av vinklene i de to hjelpetrekantene.

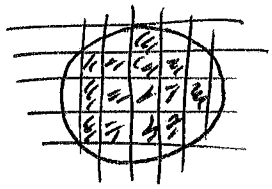
Summen er $180° + 180° = 360°$.

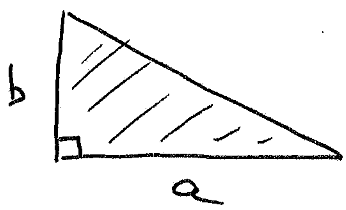
Summen av vinklene i en n-kant er $\underline{180°(n-2)}$.

4

Arealet til et rektangel med sider av lengde a og b er $a \cdot b$



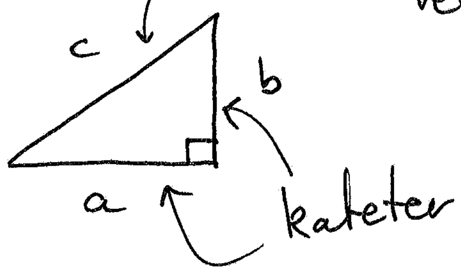




arealet er $\frac{a \cdot b}{2}$

Pytagoras sin sats $a^2 + b^2 = c^2$

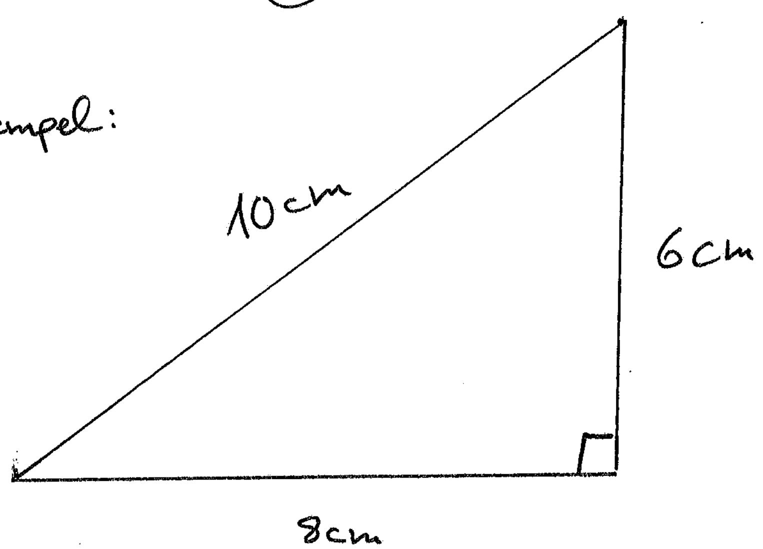
hypotenus (motsatt den rette vinkelen)



a, b lengden til katetene

c lengden til hypotenusen

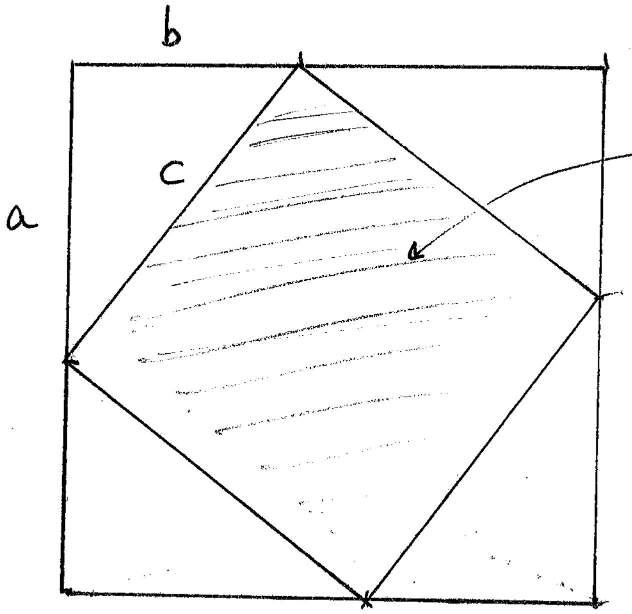
Eksempel:



$6^2 + 8^2 = 10^2$

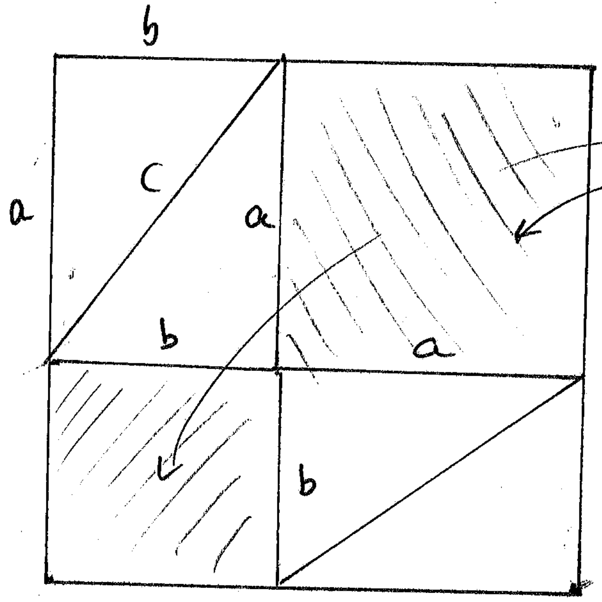
$36 + 64 = 100$

5 Geometrisk bevis for Pytagoras sin sats.



arealet er c^2

Flytter på de fire identiske trekantene inni kvadratet.



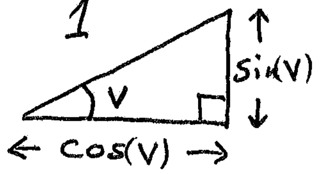
summen av arealene (utenfor trekantene) er $a^2 + b^2$

Derfor er $c^2 = a^2 + b^2$

Dette argumentet er gyldig for alle rettvinkla trekanter.

⑥ Sinus og ~~(kosinus)~~ cosinus

hypotenus har lengde 1



motstående katet

hosliggende katet

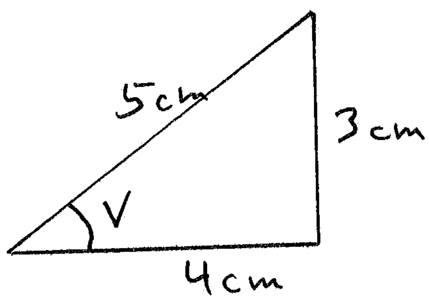
$$\sin(v) = \frac{\text{motstående katet}}{\text{hypotenus}} \qquad 0° < v < 90°$$

(enhetsløs)

$$\cos(v) = \frac{\text{hosliggende katet}}{\text{hypotenus}}$$

$\cos(0°) = 1$ $\qquad \cos(90°) = 0$

$\sin(0°) = 0$ $\qquad \sin(90°) = 1$



$$\sin(v) = \frac{3\text{cm}}{5\text{cm}} = \frac{3}{5} = 0.6$$

$$\cos(v) = \frac{4\text{cm}}{5\text{cm}} = \frac{4}{5} = 0.8$$

$\sin v$, $\cos(v)$ ligger mellom 0 og 1
for vinkler mellom $0°$ og $90°$

For alle $0 \leq x \leq 1$ så finnes det
en vinkel v mellom $0°$ og $90°$ slik
at $\sin(v) = x$

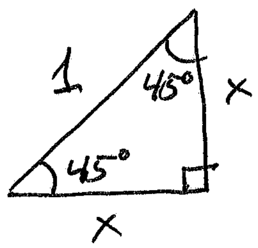
Denne vinkelen er inverssinus til x, $\sin^{-1}(x)$
alternativt: arcus sinus til x, $\underline{\arcsin(x)}$

Tilsvarende $\cos^{-1}$ og arccos.

(7) Vinkelen i eksempelet oven for er

$$\arcsin\left(\frac{3}{5}\right) = \arcsin(0.6) = 36.869\ldots^\circ$$

Eksakte verdier til sin og cos



Pytagoras:

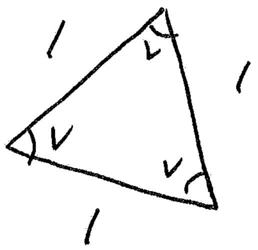
$$x^2 + x^2 = 1^2$$

$$2x^2 = 1, \quad x^2 = \frac{1}{2}$$

$x > 0$ så $x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$$\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{2}}{2}\right)$$

Så $\sin(45^\circ) = \cos(45^\circ) = \frac{1}{\sqrt{2}} \underset{(\approx)}{\simeq} \underline{0.707}$



Likesidet trekant.

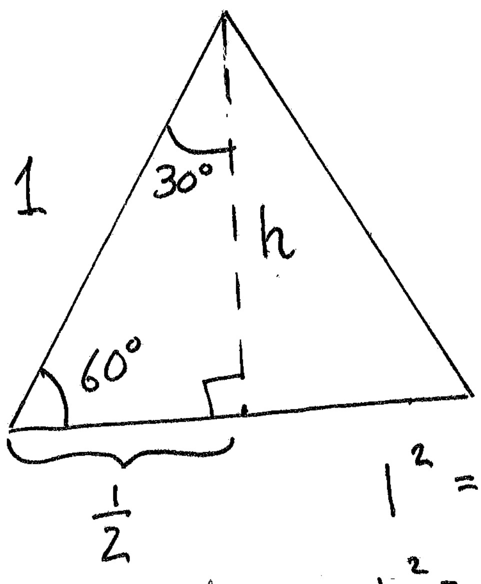
summen av vinklene

$$3 \cdot V = 180^\circ$$

$$V = 60^\circ$$

$$\cos(60^\circ) = \sin(30^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2} \approx \underline{0.866}$$



$$1^2 = \left(\frac{1}{2}\right)^2 + h^2, \quad 1 = \frac{1}{4} + h^2$$

$h^2 = \frac{3}{4}$, $h > 0$ så $h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}}$

$$\underline{h = \frac{\sqrt{3}}{2}}$$