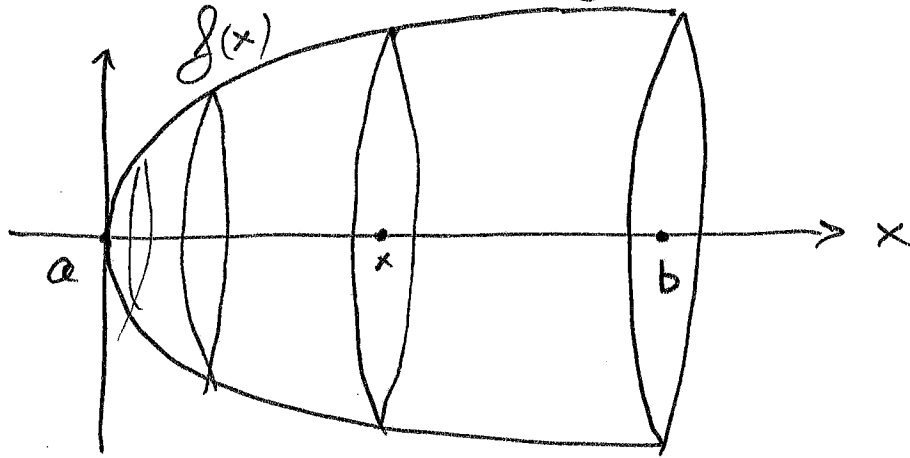


3.05.2012

Omdreiningselementer

①



Volumen $V = \int_a^b A(x) dx$

Omdreiningselementer $A(x) = \frac{\pi(f(x))^2}{}$

$$\begin{aligned} V &= \int_a^b \pi f(x)^2 dx \\ &= \pi \int_a^b f(x)^2 dx \end{aligned}$$

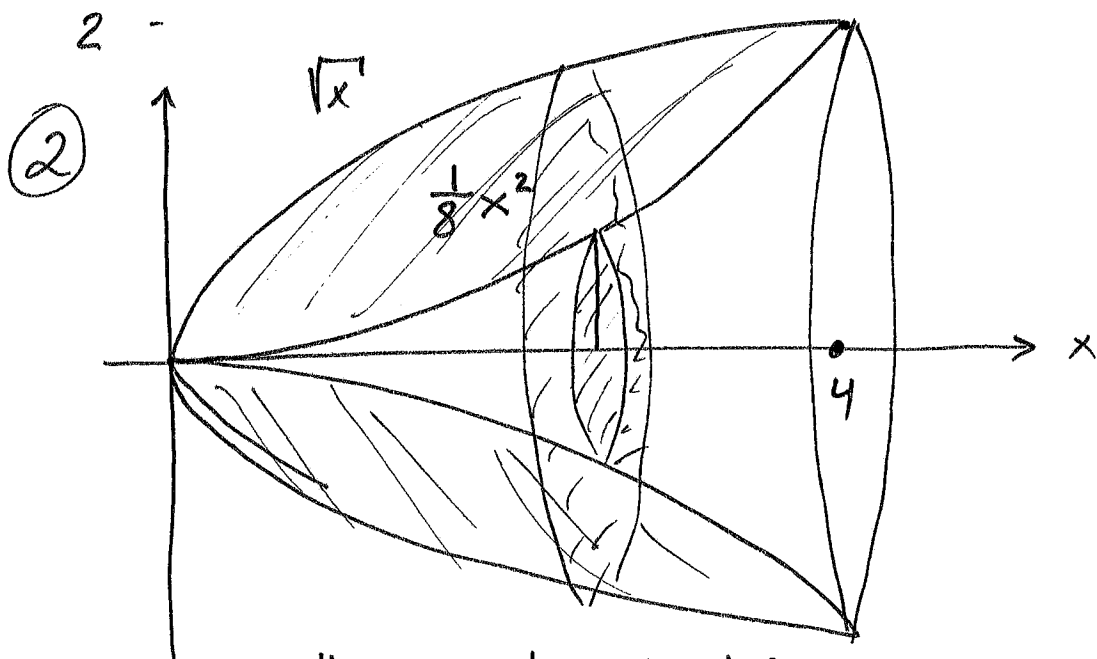
Eksempel

$f(x) = \sqrt{x}$ (figuren ovenfor)
 $a=0$ $b=4$

$(y = \sqrt{x},$
 $y^2 = x)$

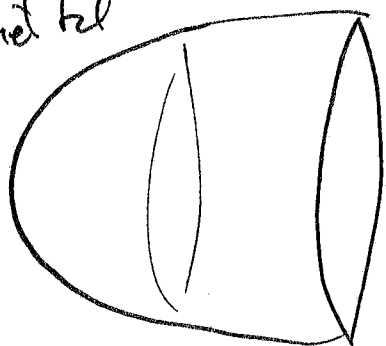
Volumen til ~~rotasjon~~ omdreiningselementer

$$\begin{aligned} \text{er } V &= \int_0^4 \pi (\sqrt{x})^2 dx \\ &= \int_0^4 \pi x dx = \pi \frac{x^2}{2} \Big|_0^4 \\ &= \underline{\underline{8\pi}} \end{aligned}$$



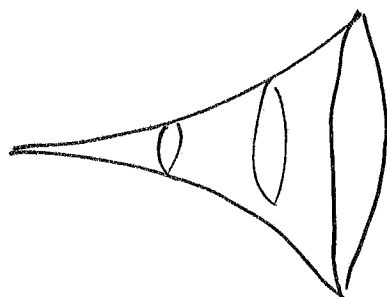
Hva er volumet til omdreininglegemet?

Volumet til



Region: R_1

Volumet til



R_2

Volumet til R_2 : $\pi \int_0^4 \left(\frac{1}{8}x^2\right)^2 dx$

$$= \pi \left(\frac{1}{8}\right)^2 \int_0^4 x^4 dx = \frac{\pi}{(2^3)^2} \frac{x^5}{5} \Big|_0^4$$

$$= \frac{\pi \cdot 4^5}{2^6 \cdot 5} = \frac{\pi \cdot 2^{10}}{2^6 \cdot 5} = \frac{\pi \cdot 2^4}{5}$$

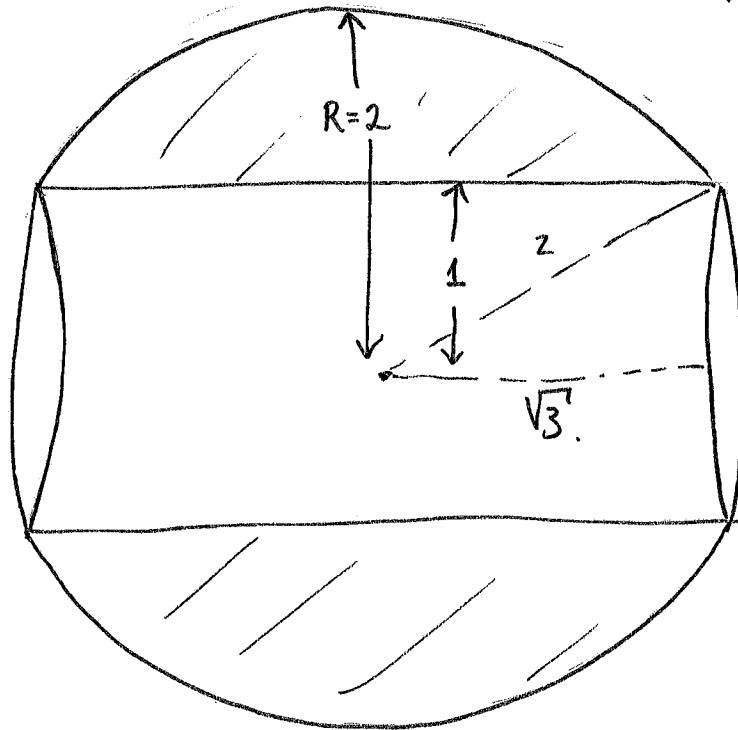
$$= \frac{16}{5} \pi$$

Volumet til omdreiningsetet er

$$V = 8\pi - \frac{16}{5}\pi = \frac{\pi}{5}(40 - 16) = \underline{\underline{\frac{24}{5} \cdot \pi \approx 15}}$$

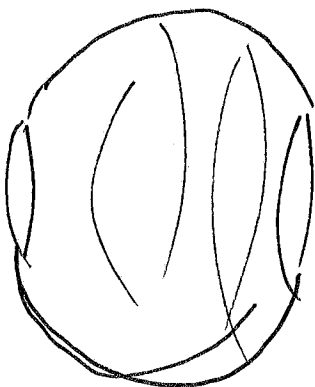
3

Finne volumet til kuleen hvor sylindereen er båret ut.



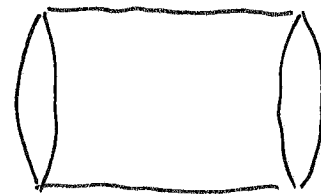
$$V = 4\pi \cdot \sqrt{3}$$

Volum til



$$6\pi \cdot \sqrt{3} = 4\pi \left(\frac{3}{2} \sqrt{3} \right)$$

Volum til

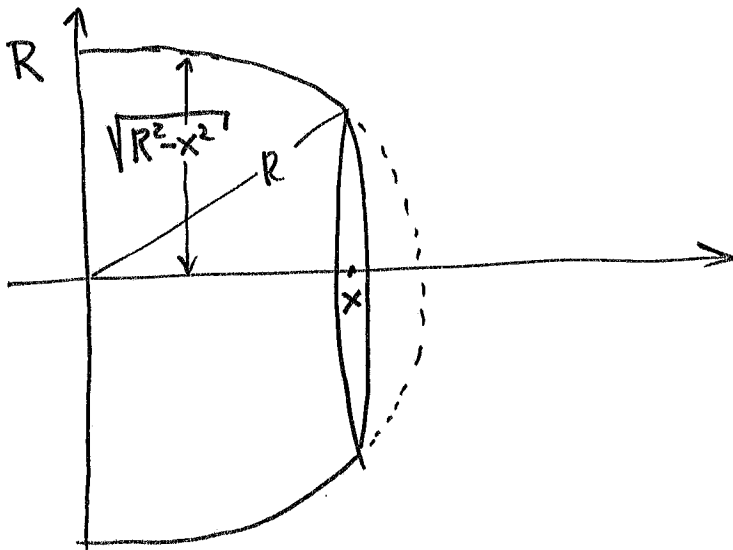


$$2 \cdot \sqrt{3} \cdot \pi = 4\pi \left(\frac{\sqrt{3}}{2} \right)$$

$$\text{Volumet er : } 4\pi \left[\frac{3}{2} \sqrt{3} - \frac{\sqrt{3}}{2} \right] = \underline{\underline{40\pi \sqrt{3}}}$$

Detaljer
→

(4)



$$0 \leq x \leq R.$$

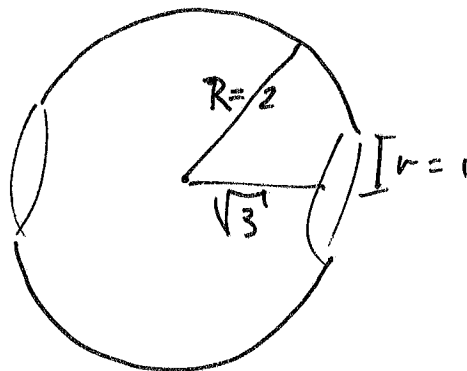
Volumet til denne halve kule arkuttet
en avstand x fra sentret er:

$$V(x) = \int_0^x \pi (\sqrt{R^2 - x^2})^2 dx$$

$$= \pi \int_0^x R^2 - x^2 dx = \pi \left(R^2 \cdot x - \frac{x^3}{3} \right) \Big|_0^x$$

$$= \pi \left(R^2 \cdot x - \frac{x^3}{3} - 0 \right) = \underline{\underline{\pi \left(R^2 \cdot x - \frac{x^3}{3} \right)}}.$$

Volumet til



$$2\pi \left(2^2 \cdot \sqrt{3} - \frac{(\sqrt{3})^3}{3} \right) = 2\pi \left(4 \cdot \sqrt{3} - \frac{3}{3} \cdot \sqrt{3} \right)$$

$$= \underline{\underline{6\pi \cdot \sqrt{3}}}.$$