16.04.2012 Substitusjon 16.1.

Minner om kjerneregelen

(1)

$$(F(u(x)))' = F'(u(x)) \cdot u'(x)$$

$$\frac{d}{dx} F(u(x)) = \frac{dF}{du}(u(x)) \cdot \frac{du}{dx}(x)$$

eksempel $\int 2x\,(1+x^2)^4\,dx$

$2x = (1+x^2)'$ La $u = x^2+1$

$$\int 2x\,(1+x^2)^4\,dx = \int u' \cdot u^4\,dx$$

$$\frac{d}{du}\left(\frac{u^5}{5}\right) = u^4$$

$$\left(\frac{(1+x^2)^5}{5}\right)' = 2x\,(1+x^2)^4.$$

Så $\int 2x(1+x^2)^4\,dx = \dfrac{(1+x^2)^5}{5} + C$

Anta $F(x)$ er en antiderivert til $f(x)$

$$(F(x))' = f(x).$$

$$(F(u(x)))' = F'(u(x)) \cdot u'(x)$$
$$= f(u(x)) \cdot u'(x)$$

$$\int f(u(x)) \cdot u'(x)\,dx = F(u(x)) + C$$
$$= \int f(u)\,du$$

Omformingen kalles substitusjon

Leibniz notasjon

(2) $\int f(u(x)) \underline{\frac{du}{dx}\, dx} = \int f(u(x))\, \underline{du}$

oppg. $\int 2x\,(x^2+7)^9\, dx$

La $u = x^2+7$ $\quad u' = 2x$

$\int u' \cdot u^9\, dx = \int u^9\, du = \frac{u^{10}}{10} + c$

$\int 2x\,(x^2+7)^9\, dx = \frac{(x^2+7)^{10}}{10} + c.$

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oppg $\int x^2 (2x^3-7)^5\, dx$

La $u = 2x^3 - 7$ $\quad u' = 2\cdot 3x^2 = 6\cdot x^2$

$\int \frac{u'}{6}\, u^5\, dx = \int \frac{1}{6}\cdot u^5\, du$

$= \frac{1}{6}\cdot\frac{u^6}{6} + c = \underline{\frac{1}{36}(2x^3-7)^6 + c}.$

oppg $\int x^3 \sin(x^4-2)\, dx$

$u = x^4 - 2$ $\quad u' = 4x^3, \quad x^3 = \frac{1}{4}u'$

$\int \frac{1}{4}u' \cdot \sin(u)\, dx = \int \frac{1}{4}\sin(u)\, du$

$= \frac{1}{4}\int \sin(u)\, du = \underline{-\frac{1}{4}\cos(x^4-2) + c}$

③ $\int \sin x \cdot (\cos x)^5 \, dx$

$u = \cos x$

$\frac{du}{dx} = u' = -\sin x$

$$\int -u' \cdot u^5 \, dx = -\int u^5 \, du$$

$$= -\frac{u^6}{6} + c$$

$$= -\frac{(\cos x)^6}{6} + c$$

Merk: $u' \, dx = \frac{du}{dx} dx \approx du.$

$$du = \frac{du}{dx} dx = (-\sin x) \, dx$$

$$-du = \sin x \, dx$$

$$\int \cos^5 x \cdot \sin x \cdot dx = \int u^5 (-du)$$

$$= -\int u^5 \, du.$$

Lineær substitusjon er substitusjon hvor

(4) $U(x) = ax + b$. $(a \neq 0)$ $du = a\,dx$

$$\int f(ax+b)\,dx = \frac{1}{a}\int f(u)\,du$$

$$= \frac{1}{a}F(ax+b) + c$$

hvor F er en antiderivert til f.

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1 - x^2$

$u' = -2x$

$du = -2x\,dx$

$$= \int \frac{1}{\sqrt{u}}\left(\frac{-1}{2}du\right)$$

$$= \frac{-1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{-1}{2}\int u^{-1/2}\,du$$

$$= \frac{-1}{2}\frac{u^{1/2}}{1/2} + c$$

$$= -u^{1/2} + c$$

$$= -(1-x^2)^{1/2} + c$$

$$= \underline{-\sqrt{1-x^2} + c}$$

eks. $\int \sin^3 x \cdot \cos^4 x \, dx$

(5) $u = \cos x \qquad \frac{du}{dx} = -\sin x$

$= \int \underbrace{\sin^2 x}_{1-\cos^2 x} \cdot \cos^4 x \cdot \sin x \, dx$

$= \int (1-u^2) \cdot u^4 \, (-du)$

$= \int (u^2-1) u^4 \, du$

$= \int u^6 - u^4 \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

opg $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

La $u = \cos x. \qquad u' = -\sin x$

$-du = +\sin x \, dx$

$\int \tan x \, dx = \int \sin x \cdot \frac{1}{\cos x} dx$

$= \int \frac{1}{u} (-du) = -\int \frac{1}{u} du$

$= -\ln|u| + c = -\ln|\cos x| + c$

oppg

(6) $\int \frac{1}{x} \ln(x^3)\, dx$

$= \int \frac{1}{x} 3 \cdot \ln x \, dx = 3 \int \frac{1}{x} \ln x \, dx$

$u = \ln x \qquad u' = \frac{1}{x} \qquad du = \frac{1}{x} dx$

$= 3 \int u' \cdot u \, dx = 3 \int u \, du$

$= 3 \frac{u^2}{2} + c$

$= \underline{\frac{3}{2} (\ln x)^2 + c}$

~~oppg~~ els

$\int 3x^2 e^{x^3} dx \qquad u = x^3 \qquad u' = 3x^2$

$= \int u' e^u dx = \int e^u du$

$= e^u + c = \underline{e^{x^3} + c}$

eks $\int x^3 (x^2-4)^8 dx$

Prøver med: $u = x^2 - 4 \qquad u' = 2x \, , \quad x^2 = u + 4$

$= \int 2x \frac{x^2}{2} (x^2-4)^8 \, dx$

$= \int u' \cdot \frac{u+4}{2} \cdot u^8 \, dx = \int \frac{u+4}{2} u^8 \, du$

$= \frac{1}{2} \int u^9 + 4u^8 \, du = \frac{1}{2}\left[\frac{u^{10}}{10} + 4 \frac{u^9}{9} \right] + c$

$\int x^3 (x^2-4)^8 dx = \underline{\frac{(x^2-4)^{10}}{20} + \frac{2(x^2-4)^9}{9} + c}$