Eksamen i	FO929A - Matematikk
Dato:	1. juni 2011
Talet på oppgåver:	5
Vedlegg:	Formelsamling
Hjelpemiddel:	Kalkulator

Ein skal grunngi alle svar. Alle deloppgåver har lik vekt

Oppgåve 1
Løys desse likningane:

a) $7\sin x - 5 = 0, \quad x \in [0, 2\pi\rangle$

b) $x^2 - 2x + 1 = 9$

c) $\ln(x+1) - \ln(x-1) = 1$

d) $7^{x^2+x} = 1$

Oppgåve 2
Deriver desse funksjonane:

a) $f(x) = x^{19} + \frac{5}{3x^2} + 2x\sqrt[3]{x}$

b) $g(x) = x^2e^{3x} + \pi$

c) $h(x) = \ln\left(4 \cdot \frac{x-1}{x^2+3x}\right)$

Rekn ut desse bestemte og ubestemte integrala:

d) $\int \left(-7x^{-2,25} - 3x^{-1} + \frac{2}{\sqrt{x}}\right) dx$

e) $\int \frac{3\sin x}{\cos^3 x}\, dx$

f) $\int_0^{\pi} t\sin(2t)\, dt$

22.03.2012

(1)

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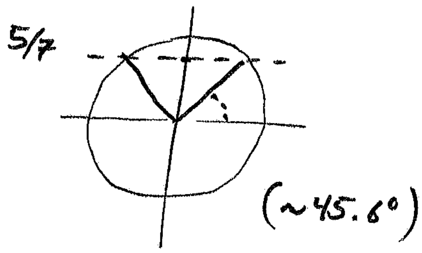
1 a) $7\sin x - 5 = 0$

$$\frac{7\sin x}{7} = \frac{5}{7}$$

$$\sin x = 5/7$$



$$x = \arcsin(5/7) \left(= \sin^{-1}(5/7)\right) = \underline{0.7956}$$

$$\text{og } x = \pi - \arcsin(5/7) = \underline{2{,}346}$$

b) $x^2 - 2x + 1 = 9$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

Løsningene er $\underline{x = -2}$ og $\underline{x = 4}$

c) $\ln(x+1) - \ln(x-1) = 1$ $\quad x > 1$

$$\left(\ln a + \ln b = \ln(a \cdot b) . \quad r \cdot \ln a = \ln a^r \right)$$

$$\ln(x+1) + \ln(x-1)^{-1} = 1$$

$$e^{\ln\left((x+1)\cdot(x-1)^{-1}\right)} = e^1$$

$$\frac{x+1}{x-1} = e$$

$$x+1 = e(x-1) = e \cdot x - e$$

$$(e-1)\cdot x = 1+e$$

② $$x = \frac{e+1}{e-1} \qquad (>1)$$

d) $$7^{x^2+x} = 1$$

$$7^0 = 1$$

$$7^{x^2+x} = 7^0$$

Så $$x^2 + x = 0$$

$$x(x+1) = 0$$

Løsningene er $\underline{x=-1}$ og $\underline{x=0}$.

③

Oppsummering

Derivasjon

Definisjone. $\frac{d}{dx}f(x) = f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

Derivasjonsreglene

Derivasjon er linieær

$$(f(x) + g(x))' = f'(x) + g'(x)$$

$$(k \cdot f(x))' = k \cdot f'(x)$$

k konstant.

Kjerneregelen $(f(U(x)))' = f'(U(x)) \cdot U'(x)$

Produktregelen $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$(x^r)' = r x^{r-1}$ r reelt tall $\left(\begin{array}{l} \frac{1}{x} = x^{-1} \\ \sqrt[n]{x} = x^{1/n} \end{array} \right)$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(e^x)' = e^x$$

$$(\ln|x|)' = \frac{1}{x}$$

$$a^x = (e^{\ln a})^x = e^{x \cdot \ln a}$$

$$\log_a x = \frac{\ln x}{\ln a} \quad \begin{array}{l} a > 0 \\ a \neq 1. \end{array}$$

(4)

2 a) $f(x) = x^{19} + \frac{5}{3x^2} + 2x\sqrt[3]{x}$

$$f'(x) = (x^{19})' + \frac{5}{3}\left(\frac{1}{x^2}\right)' + 2\left(x\cdot\sqrt[3]{x}\right)'$$

$$\frac{1}{x^2} = x^{-2},$$

$$x\cdot\sqrt[3]{x} = x\cdot x^{1/3} = x^1\cdot x^{1/3} = x^{1+1/3} = x^{4/3}.$$

$$f'(x) = (x^{19})' + \frac{5}{3}(x^{-2})' + 2(x^{4/3})'$$

$$= 19\cdot x^{18} + \frac{5}{3}(-2\cdot x^{-3}) + 2\cdot\frac{4}{3}x^{1/3}$$

$$= \underline{19x^{18} + \frac{-10}{3x^3} + \frac{8}{3}\sqrt[3]{x}}$$

(π konstant)

b) $g(x) = x^2 e^{3x} + \pi$

$$g'(x) = (x^2)'e^{3x} + (x^2)(e^{3x})' + (\pi)'$$

$$= 2xe^{3x} + x^2\left(e^{3x}\cdot(3x)'\right) + 0$$

$$= 2xe^{3x} + 3x^2e^{3x}$$

$$= (2x+3x^2)e^{3x}$$

$$= \underline{x(2+3x)e^{3x}}$$

⑤ c) $h(x) = \ln\left(4 \cdot \frac{x-1}{x^2+3x}\right)$

$= \ln 4 + \ln\left((x-1) \cdot (x^2+3x)^{-1}\right)$

$= \ln 4 + \ln(x-1) - \ln(x^2+3x)$

$= \ln 4 + \ln(x-1) - \left(\ln(x) + \ln(x+3)\right)$

$h'(x) = (\ln 4)' + (\ln(x-1))' - (\ln(x))' - (\ln(x+3))'$

$= 0 + \frac{(x-1)'}{(x-1)} - \frac{1}{x} - \frac{(x+3)'}{x+3}$

$= \underline{\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x+3}}$

Alternativt

$(\ln 4)' + (\ln(x-1))' - (\ln(x^2+3x))'$

$= 0 + \frac{1}{x-1} - \frac{(x^2+3x)'}{x^2+3x}$

$= \frac{1}{x-1} - \frac{2x+3}{x^2+3x}$

$= \frac{x^2+3x-(2x+3)(x-1)}{(x-1)(x^2+3x)}$

$= \frac{x^2+3x-[2x^2+x-3]}{(x-1)(x^2+3x)}$

$= \underline{\frac{-x^2+2x+3}{x(x+3)(x-1)}} = \underline{\frac{-(x-3)(x+1)}{x(x+3)(x-1)}}$