1

11.6 og 11.8

$$(\ln|x|)' = \frac{1}{x} \qquad x \neq 0$$

$\ln = \log_e$. logaritme med basis e, naturlig logaritme.

$$\log_a(x) = \frac{\ln x}{\ln a}$$

(Vi vet at $\log_a x = k \cdot \ln x$ k konstant.

Setter $x=a$: $\underbrace{\log_a a}_{1} = k \cdot \ln a$

$1 = k \cdot \ln a$

$k = \frac{1}{\ln a}$)

$$\frac{d}{dx}\log_a(x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\frac{d}{dx}\ln(x)$$

$$= \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\boxed{\frac{d}{dx}\log_a(|x|) = \frac{1}{(\ln a)\,x} \qquad x \neq 0.}$$

eks

$f(x) = \ln(2x+1)$ (definert for $x > \frac{-1}{2}$)

$\frac{d}{dx}\ln(2x+1) = \frac{d\ln(u)}{du} \cdot \frac{du}{dx}$ kjerneregelen ($u = 2x+1$)

$= \frac{1}{u} \cdot (2x+1)' = \underline{\frac{2}{2x+1}}$

eks $\quad f(x) = \ln(x^7)$

2

Som sammensatt funksjon: $\quad u = x^7$

$$\frac{d}{dx}\ln(x^7) = \frac{d\ln(u)}{du}\cdot\frac{dx^7}{dx}$$

$$= \frac{1}{u}\cdot 7x^6 = \frac{7\cdot x^6}{x^7}$$

$$= \underline{\frac{7}{x}}$$

alternativt: $\quad \ln(x^7) = 7\ln x$

$$\frac{d}{dx}\ln(x^7) = \frac{d}{dx}7\ln x = 7\frac{d\ln x}{dx}$$

$$= \underline{\frac{7}{x}}$$

eks $\quad f(x) = (\ln x)^4 \qquad$ lar $u = \ln x$

$$\frac{d}{dx}(\ln x)^4 = \frac{du^4}{du}\cdot\frac{du}{dx}$$

$$= 4u^3\cdot\frac{1}{x}$$

$$= \underline{4(\ln x)^3\cdot\frac{1}{x}}$$

Oppg. Deriver $\quad f(x) = \left(\ln((x+1)^4)\right)^2$

$$f'(x) = 2\left(\ln((x+1)^4)\right)\frac{d}{dx}\left(\ln((x+1)^4)\right)$$

$$= 2\ln((x+1)^4)\frac{1}{(x+1)^4}\cdot\underbrace{\frac{d}{dx}(x+1)^4}_{4(x+1)^3(x+1)'}$$

$$= \underline{2\ln((x+1)^4)\cdot\frac{4(x+1)^3}{(x+1)^4}}$$

alternativt:

(3)

$$f(x) = (4\ln(x+1))^2 = 4^2(\ln(x+1))^2$$
$$= 16(\ln(x+1))^2$$

$$f'(x) = 16\cdot 2(\ln(x+1))\cdot(\ln(x+1))'$$
$$= \underline{\frac{32\ln(x+1)}{x+1}}$$

eks

$$\cos(\ln|x|)$$

$$\frac{d}{dx}\cos(\ln|x|) = -\sin(\ln|x|)\cdot(\ln|x|)'$$
$$= \underline{-\sin(\ln|x|)\cdot\frac{1}{x}}.$$

oppg. Deriver $2-\ln(|\cos x|)$

$$(2-\ln|\cos x|)' = -(\ln(|\cos x|))'$$

$u = \cos x$

$$= -\frac{d\ln|u|}{du}\cdot\frac{d\cos x}{dx}$$

$$= -\frac{1}{u}\cdot(-\sin x)$$

$$= \frac{-1}{\cos x}\cdot(-\sin x)$$

$$= \frac{\sin x}{\cos x} = \underline{\tan x}$$

eks (4)

$$f(x) = x\ln(|x|) - x$$

Deriver $f(x)$

$$f'(x) = (x\ln(|x|))' - (x)'$$ produktregel

$$= (x)' \cdot \ln(|x|) + x \cdot (\ln|x|)' - 1$$

$$= 1 \cdot \ln|x| + x \cdot \frac{1}{x} - 1$$

$$= \ln|x| + 1 - 1$$

$$= \underline{\underline{\ln|x|}}$$

oppg. Deriver $x \cdot \ln(\overbrace{\sqrt{x+1}}^{(x+1)^{1/2}}) + \ln 5$

(benytter $\ln x^r = r\ln x$) $= \frac{x}{2} \cdot \ln(x+1) + \ln 5$

$$\left(\frac{x}{2}\ln(x+1) + \ln 5\right)' = \frac{1}{2}(x\ln(x+1))' + \underbrace{(\ln 5)'}_{=0}$$

$$= \frac{1}{2}\left[(x)' \cdot \ln(x+1) + x \cdot (\ln(x+1))'\right] + 0$$

$$= \frac{1}{2}\left[1 \cdot \ln(x+1) + x \cdot \frac{1}{x+1}(x+1)'\right]$$

$$= \underline{\frac{1}{2}\left(\ln(x+1) + \frac{x}{x+1}\right)}$$

(5)

oppg. Deriver $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

Husk logaritme reglene

$\ln(a \cdot b) = \ln a + \ln b$

$\ln a^r = r \ln a$

$\ln\left(\frac{a}{b}\right) = \ln(a \cdot b^{-1})$

$= \ln a + \ln(b^{-1})$

$= \ln a - \ln b$

$$\left(\ln(1+x) - \ln(1-x)\right)'$$

$$= \frac{1}{1+x}\underbrace{(1+x)'}_{1} - \frac{1}{1-x}\underbrace{(1-x)'}_{-1}$$

$$= \frac{1}{1+x} + \frac{1}{1-x}$$

$$= \frac{(1-x)}{(1+x)(1-x)} + \frac{(1+x)}{(1-x)(1+x)}$$

$$= \frac{1-x+1+x}{1-x^2}$$

$$= \underline{\underline{\frac{2}{1-x^2}}}$$

⑥ $(e^x)' = e^x$ $\qquad a = e^{\ln a}$ $\quad a>0.$

oppg. Deriver $\left(\frac{1}{2}\right)^x$

$$\left(\frac{1}{2}\right)^x = \left(e^{\ln(\frac{1}{2})}\right)^x = e^{x\cdot\ln(1/2)}$$

$$\frac{d}{dx}\left(\frac{1}{2}\right)^x = \frac{d}{dx} e^{x\cdot\ln(1/2)}$$

$$= \frac{d e^u}{du}\cdot\frac{d}{dx}(x\cdot\ln(1/2))$$

$$= e^u\cdot\ln(1/2)$$

$$= e^{x\cdot\ln 1/2}\cdot\ln(1/2)$$

$$= \left(\frac{1}{2}\right)^x\ln(1/2)$$

$$= \underline{-\ln(2)\cdot\left(\frac{1}{2}\right)^x}$$

oppg Deriver $e^{2\ln x + 1}$.

(Hint $e^{\ln x} = x$)

$$e^{2\ln x + 1} = e^{2\ln x}\cdot e^1$$

$$= \left(e^{\ln x}\right)^2\cdot e$$

$$= x^2\cdot e$$

$$\left(e^{2\ln x+1}\right)' = e\cdot(x^2)' = \underline{2x\cdot e} = \underline{\underline{(2e)x}}$$