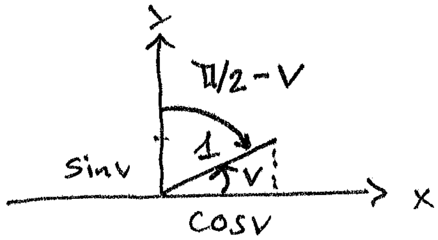
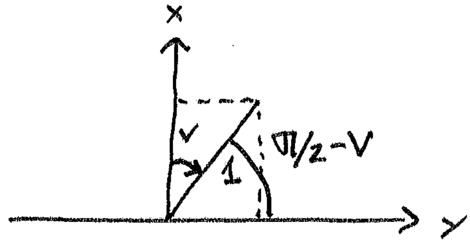
9.02.2012

10.5 Kosinus

(1) Sist gang viste vi at $\frac{d}{dx}\sin x = \cos x$





$$\cos v = \sin\left(\frac{\pi}{2} - v\right)$$

(syne = vise)

$$\sin v = \cos\left(\frac{\pi}{2} - v\right)$$

Vi viser at $\frac{d}{dx}\cos x = -\sin x$

(ved å bruke $(\sin(x))' = \cos x$)

$$\begin{aligned}
\frac{d}{dx}\cos x &= \frac{d}{dx}\sin\left(\frac{\pi}{2} - x\right) \\
&= \cos\left(\frac{\pi}{2} - x\right)\cdot\left(\frac{\pi}{2} - x\right)' \\
&= -\cos\left(\frac{\pi}{2} - x\right) \\
&= \underline{-\sin x}
\end{aligned}$$

bevis fra definisjonen av den deriverte:

Addisjonsformelen for cos: $\cos(x+h) = \cos x \cos h - \sin h \sin x$

$$\begin{aligned}
(\cos x)' &= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} \\
&= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \sin h \cdot \sin x}{h} \\
&= \cos x \lim_{h\to 0}\frac{\cos h - 1}{h} - \sin x \cdot \lim_{h\to 0}\frac{\sin h}{h} \\
&= \cos x \cdot 0 - \sin x \cdot 1 \\
&= \underline{-\sin x}
\end{aligned}$$

② Eksempel

$$\frac{d}{dx}\cos(-2x+1) \overset{\text{kjerneregel}}{=} -\sin(-2x+1)\ (-2x+1)'$$
$$= -\sin(-2x+1)\ (-2)$$
$$= \underline{2\sin(-2x+1)}.$$

Deriver $\sqrt[4]{x}\cdot\cos(\pi x)$.

$$\sqrt[4]{x} = x^{1/4}$$

$$\left(x^{1/4}\cdot\cos(\pi x)\right)' \overset{\text{produktregel}}{=} (x^{1/4})'\cos(\pi x) + (x^{1/4})\cdot(\cos(\pi x))'$$
$$= \left(\frac{1}{4}x^{\frac{1}{4}-1}\right)\cdot\cos(\pi x) + (x^{1/4})\cdot\left(-\sin(\pi x)\cdot(\pi x)'\right)$$
$$= \frac{1}{4}x^{\frac{1}{4}-1}\cdot\cos(\pi x) + x^{1/4}\left(-\sin(\pi x)\cdot\pi\right)$$
$$= \frac{1}{4x^{3/4}}\left[\cos(\pi x) + 4x^{3/4}\cdot x^{1/4}(-\pi\sin(\pi x))\right]$$
$$= \frac{1}{4x^{3/4}}\left[\cos(\pi x) - 4\pi\cdot x\sin(\pi x)\right]$$
$$= \underline{\frac{\cos(\pi x) - 4\pi x\sin(\pi x)}{4x^{3/4}}}$$

Deriver $\cos(\pi\cdot\cos x)$

$$\left(\cos(\pi\cos x)\right)' = -\sin(\pi\cdot\cos x)\cdot(\pi\cdot\cos x)'$$
$$= -\sin(\pi\cdot\cos x)\cdot\pi\cdot(-\sin x)$$
$$= \underline{\pi\cdot\sin x\cdot\sin(\pi\cos x)}$$

③ Deriver $\cos\left(\frac{x+1}{x}\right)$.

Merk at $\frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x} = 1 + x^{-1}$

$$\left(\cos\left(\frac{x+1}{x}\right)\right)' = \left(\cos\left(1+x^{-1}\right)\right)'$$

$$= -\sin\left(1+x^{-1}\right)\cdot\left(1+x^{-1}\right)'$$

$$= -\sin\left(1+x^{-1}\right)\cdot\left(-1\cdot x^{-1-1}\right)$$

$$= \underline{\frac{1}{x^2}\cdot\sin\left(1+\frac{1}{x}\right)}.$$

oppgave Deriver: $\cos(2x) - 2\cos^2 x$

Notasjon: $\cos^2 x = (\cos x)^2$ $\Big/\!\!\Big/ \left(\frac{d\cos^2 x}{dx} = \frac{d(\cos x)^2}{d\cos x}\cdot\frac{d\cos x}{dx}\right)$

$$\left(\cos(2x) - 2\cos^2 x\right)' = \left(\cos(2x)\right)' - 2\left(\cos^2 x\right)'$$

$$= -\sin(2x)\cdot(2x)' - 2(2\cdot\cos x)\cdot(\cos x)'$$

$$= -\sin(2x)\cdot 2 - 4\cos x\cdot(-\sin x)$$

$$= -2\sin 2x + 4\sin\cdot\cos x$$

$$= 2\left(2\sin x\cdot\cos x - \sin 2x\right)$$

$$= 2\left(\sin 2x - \sin 2x\right) = 0$$

(siden $\sin 2x = 2\sin x\cdot\cos x$)

Er $\cos 2x - 2\cos^2 x$ konstant?

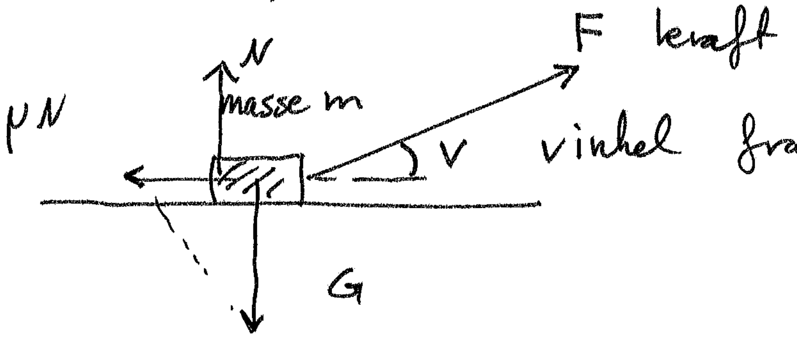
$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x - 2\cos^2 x = \cos^2 x - \sin^2 x - 2\cos^2 x$$

$$= -\cos^2 x - \sin^2 x = -(\cos^2 x + \sin^2 x)$$

$$= \underline{\underline{-1}}$$ ja!

④ Fysikk eksempel.



Statisk friksjons koeffisient $\mu = 0.5$

masse $m = 30$ kg

Hvilke vinkel V krever minst kraft for å sette legemet i bevegelse?

$$N + F \cdot \sin V = m \cdot g$$

$$\mu \cdot N = \mu (m \cdot g - F \cdot \sin V)$$

$$= F \cdot \cos V$$ (giv bevegelse)

(horisontal komponenten til $\vec{F}$).

$$F \cos V = \mu (m \cdot g - F \sin V)$$

$$F (\cos V + \mu \cdot \sin V) = \mu m \cdot g$$

$$F = \frac{\mu m g}{\cos V + \mu \cdot \sin V}.$$

F er minst mulig når nevneren $\cos V + \mu \sin V$ er størst mulig.

$$(\cos V + \mu \sin V)' = -\sin V + \mu \cos V$$

Den deriverte til $\cos V + \mu \sin V$ er 0 når

$$\tan V = \frac{\sin V}{\cos V} = \mu.$$

(5) $\cos v + \mu \sin v$ har ét toppunkt når $\tan v = \mu$

siden: $(\cos v + \mu \sin v)'' = -(\cos v + \mu \sin v) < 0$.

Setter vi inn tallverdiene får vi:

$\tan v = \frac{1}{2}$: $v = 26.5°$

Kraften som trengs er da $\underline{131.5N}$

Hvis vi hadde dratt boksen horisontalt $(v=0)$

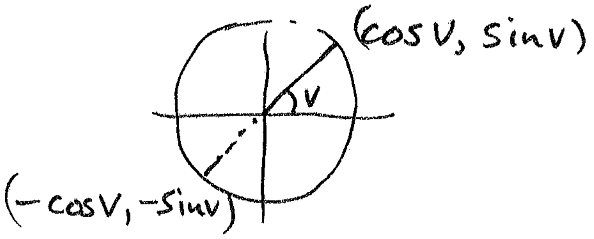
måtte vi brukt $\underline{147N}$

10.6 Tangensfunksjonen

$\tan x = \dfrac{\sin x}{\cos x}$ definert når $\cos x \neq 0$

$x \neq \frac{\pi}{2} + \pi \cdot n$ n heltall.

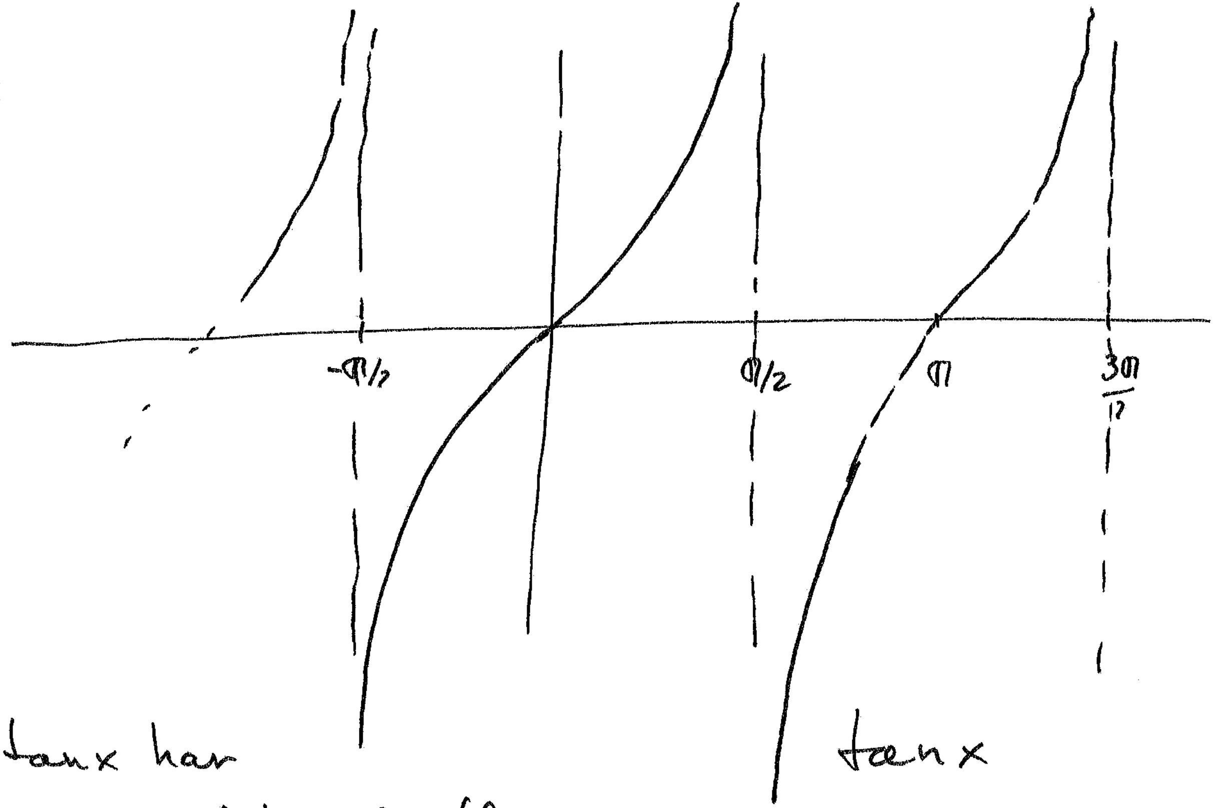
$\tan(x+\pi) = \tan x$ perioden er π



$$\frac{-\sin v}{-\cos v} = \frac{\sin v}{\cos v}$$

$\tan(v+\pi) = \tan v$

⑥



tan x har ingen ekstremalpunkt.

$$\frac{d}{dx}\tan x = 1 + \tan^2 x = \frac{1}{\cos^2 x} = \sec^2 x$$

Vi viser dette:

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{(\sin x)' \cdot \cos x - \sin x (\cos x)'}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \left(\frac{\sin x}{\cos x}\right)^2 = 1 + \tan^2 x$$

alternativt bruker vi Pytagoras sin sats:

$$\frac{d}{dx}\tan x = \frac{(\sin^2 x + \cos^2 x)}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \left(\frac{1}{\cos x}\right)^2 = (\sec x)^2$$

(hvor $\sec x = \frac{1}{\cos x}$)

Hva er $\frac{d^2}{dx^2} \tan x$?

7 $(\tan x)'' = (\tan^2 x + 1)'$

$= ((\tan x)^2)'$

$= 2(\tan x)(\tan x)'$

$= 2\tan x\,(1 + \tan^2 x)$

tan x har vendepunkt i sine nullpunkt

tan x er konkav opp når den er positiv

——— ned ——— negativ.

eksempel $(\tan(2x-3))'$ kjerneregelen

$= (1 + \tan^2(2x-3))(2x-3)'$

$= \underline{2(1 + \tan^2(2x-3))}$

oppgave Deriver $\frac{1}{\tan^3 x}$

$\frac{1}{\tan^3 x} = \frac{1}{(\tan x)^3} = (\tan x)^{-3}$

$\tan x = u$

$\frac{d}{dx}\left(\frac{1}{\tan^3 x}\right) = \frac{d}{dx}(\tan x)^{-3}$

$= \frac{d}{du} u^{-3} \cdot \frac{du}{dx}$

$= -3u^{-3-1} \cdot (\tan x)'$

$= \frac{-3}{(\tan x)^4} \cdot (1 + \tan^2 x) = \underline{\frac{-3(1+\tan^2 x)}{\tan^4 x}}$