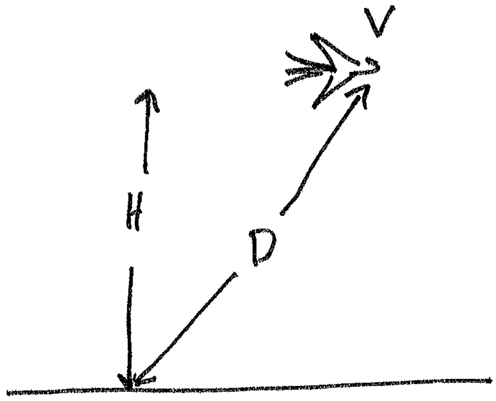
1



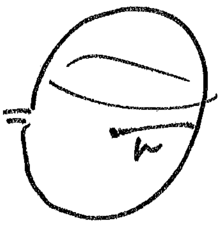
rett strekning.

t : tiden

Hva er $\frac{dD}{dt}$ uttrykt ved D, H og farten V til flyet.

2

$\frac{dV}{dt}$ $\rightarrow$



Hva er $\frac{dr}{dt}$?

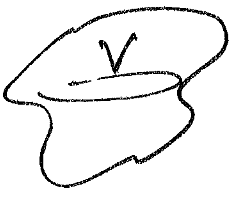
3 $\quad T$ temperatur

$$\frac{1}{x}\cdot\frac{dx}{dT} = \sigma$$

areal A

Hva er $\frac{1}{A}\frac{dA}{dT}$?

Volum V

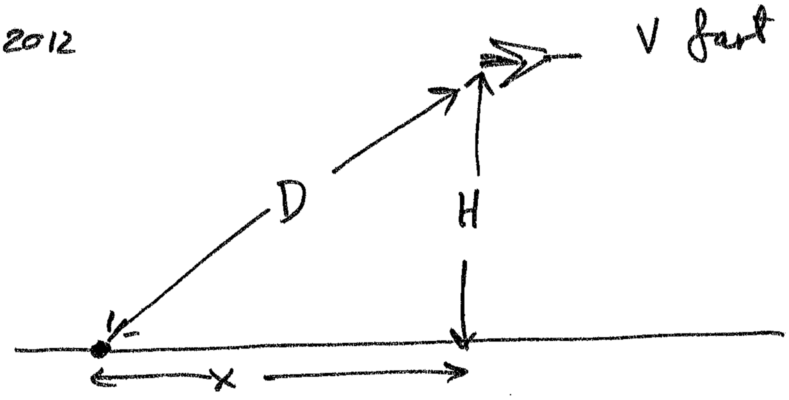


Hva er $\frac{1}{V}\frac{dV}{dT}$?

2.02.2012

Eks

1



$H = 3000$ m

$V = 800 \text{km/t} = \frac{800}{3.6}$ m/s $= \frac{200}{0.9}$ m/s

$= 200$ m/s $(0,111\ldots) = \underline{222 \text{ m/s}}$

Hva er vekstfarten $\frac{dD}{dt}$ når $D = 5{,}000$ km?

$x^2 + H^2 = D^2$ Pytagoras.

Farten til flyet $\underline{V = \frac{dx}{dt}}$

$$\left(\frac{d}{dt} D(t) = \lim_{\Delta t \to 0} \frac{D(t+\Delta t) - D(t)}{\Delta t} \ldots \right)$$

$$\frac{d}{dt}(x^2 + H^2) = \frac{d}{dt} D^2$$

$$\frac{d(x^2)}{dx} \cdot \frac{dx}{dt} + 0 = \frac{dD^2}{dD} \cdot \frac{dD}{dt} \quad \text{(H konstant)}$$

$$2x \cdot \frac{dx}{dt} = 2D \cdot \frac{dD}{dt}$$

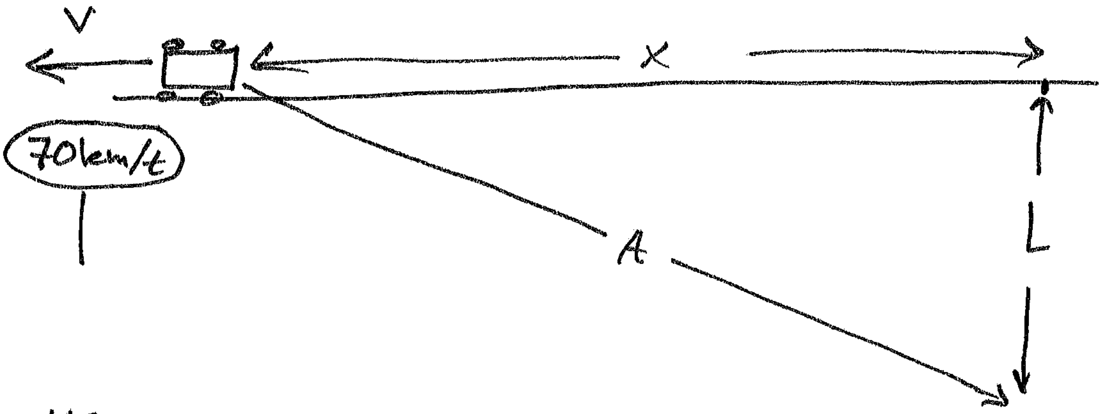
Så $\underline{\frac{dD}{dt} = \frac{x}{D} \frac{dx}{dt}}$

$$\frac{dD}{dt} = \frac{\sqrt{D^2 - H^2}}{D} \frac{dx}{dt} = \frac{4000 \text{m}}{5000 \text{m}} 222 \text{m/s} = \frac{4}{5} \cdot 222 \text{m/s}$$

$$= \underline{177{,}6 \text{ m/s}}$$

oppg

2



$L = 40m$

Dere måler $\frac{dA}{dt} = 63 km/t$ og $A = 100 m$.

Kjører bilen for fort? Hvor fort kjører bilen?

—

Pytagoras $A^2 = L^2 + x^2$ L konstant

(Den deriverte til $(A(t))^2$ med hensyn til t) $\frac{d}{dt}(A^2) = \frac{dA^2}{dA} \cdot \frac{dA}{dt} = 2A(t) \cdot \frac{dA}{dt}$

$(A^2_{(t)})' = 2A(t) \cdot A'(t)$

$\frac{d}{dt} x^2 = 2x(t) \frac{dx}{dt}$

$\frac{d}{dt} A^2 = \frac{d}{dt}(L^2 + x^2)$

$2A \cdot \frac{dA}{dt} = 2x \frac{dx}{dt}$

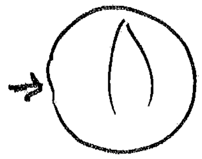
Farten til bilen $v = \frac{dx}{dt} = \frac{A}{x} \frac{dA}{dt}$

$$V = \frac{100m}{\sqrt{(100m)^2 - (40m)^2}} \cdot 63 km/t$$

$$= \underline{68.7 \; km/t}$$

Bilen kjører under fartsgrensen.

③ Vi blåser opp en ballong



Anta at $\frac{dV}{dt} = 40\,cm^3/s$ hvor

V er volumet til ballongen.

Hva er vekstfarten til radiusen r når $r = 10cm$?

—

Volumet til en kule med radius r er

$$V = \frac{4\pi r^3}{3}.$$

$$\frac{dV}{dt} = \frac{d(4\pi r^3/3)}{dr} \cdot \frac{dr}{dt} \qquad \text{(kjerneregelen)}$$

$$\left(V'_{(t)} = \left(\frac{4\pi r^3}{3}\right)' \cdot r'_{(t)} \right)$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Så $$\frac{dr}{dt} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt}$$

$$= \frac{1}{4\pi (10cm)^2} \cdot 40\,cm^3/s$$

$$= \frac{40}{4\pi \cdot 100} \frac{cm^3/s}{cm^2} = \frac{1}{\pi \cdot 10}\,cm/s$$

$$\frac{dr}{dt} = \underline{0{,}0318\,cm/s}$$

x

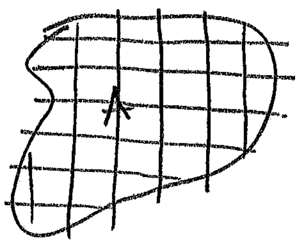
eks

(4) lengden øker når temperaturen øker

$$\frac{1}{x}\frac{dx}{dT} = \sigma \quad \text{(utvidelses koeffisient)}$$

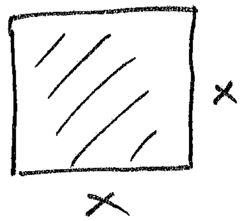
(σ sigma)

Σ Sigma



Hva er $\frac{1}{A}\frac{dA}{dT}$?

Tilfelle når vi har et kvadrat

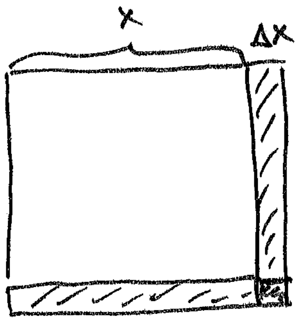


$$A = x^2$$

$$\frac{dA}{dT} = \frac{dx^2}{dT} \overset{\text{kj. regel}}{=} \frac{dx^2}{dx}\cdot\frac{dx}{dT} = 2x \cdot \frac{dx}{dT}$$

Så $$\underline{\frac{1}{A}\cdot\frac{dA}{dT}} = \frac{2x}{x^2}\frac{dx}{dT} = 2\cdot\frac{1}{x}\frac{dx}{dT}$$

$$= \underline{\underline{2\cdot\sigma}}$$

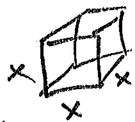


$$\Delta A = \Delta x \cdot x + \Delta x \cdot \Delta x + x \cdot \Delta x$$

$$= \Delta x(2x) + (\Delta x)^2$$

$$\frac{\Delta A}{\Delta T} = \frac{\Delta x}{\Delta T}(2x) + \Delta x \cdot \frac{\Delta x}{\Delta T}$$

Volum : $V = x^3$



$$\frac{1}{V}\frac{dV}{dT} = \frac{1}{x^3}\cdot\frac{dx^3}{dx}\cdot\frac{dx}{dT} = \frac{3x^2}{x^3}\frac{dx}{dT} = 3\cdot\frac{1}{x}\frac{dx}{dT}$$

$$= \underline{3\sigma} \quad , \quad \underline{\frac{dV}{dT} = 3\sigma\cdot V}$$