26 jan 2012

9.9 Kvotientregelen

(1) $\frac{f(x)}{g(x)}$ kvotient.

$f(x) = \frac{1}{x} = x^{-1}$ $\qquad f'(x) = -1 \cdot x^{-1-1} = -x^{-2} = \underline{\frac{-1}{x^2}}$

$g(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$

Bruker kjerneregelen (med kjerne $u = 1+x^2$)

$g(x) = u^{-1}$ $\qquad g'(x) = (u^{-1})' \cdot u'(x)$

$$= \frac{-1}{u^2} \cdot (1+x^2)'$$

$$= \underline{\frac{-2x}{(1+x^2)^2}}$$

$$\left(\frac{1}{g(x)}\right)' = \left((g(x))^{-1}\right)' = \frac{-1}{(g(x))^2} \cdot g'(x)$$

$$= \underline{\frac{-g'(x)}{(g(x))^2}}$$

Eks $f(x) = \frac{1}{(x^2+1)^7} = (x^2+1)^{-7}$ $\qquad$ kjerne x^2+1

$f'(x) = (u^{-7})' \cdot u'(x) = -7u^{-8}(x^2+1)'$

$$= \frac{-7(2x)}{(x^2+1)^8} = \underline{\frac{-14x}{(x^2+1)^8}}$$

Derivasjon utført ved å bruke $\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{(g(x))^2}$

$$\left(\frac{1}{(x^2+1)^7}\right)' = \frac{-((x^2+1)^7)'}{((x^2+1)^7)^2} = -\frac{7(x^2+1)^6 \cdot 2x}{(x^2+1)^{14}}$$

$$= \frac{-14x \; (x^2+1)^6}{(x^2+1)^8 \cdot (x^2+1)^6} = \underline{\frac{-14x}{(x^2+1)^8}}$$

Kvotientregelen :

② $$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

Bevis : $$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x)\cdot\frac{1}{g(x)}\right)'$$ produktregelen

$$= f'(x)\cdot\frac{1}{g(x)} + f(x)\cdot\left(\frac{1}{g(x)}\right)'$$

$$= f'(x)\cdot\frac{1}{g(x)} + f(x)\,\frac{-g'(x)}{(g(x))^2}$$

$$= \frac{f'(x)\cdot g(x)}{(g(x))^2} - \frac{f(x)\cdot g'(x)}{(g(x))^2}$$

$$= \frac{f'(x)\cdot g(x) - f(x)\,g'(x)}{(g(x))^2}.$$

Eks $$\left(\frac{x^2}{x^3-8}\right)' = \frac{(x^2)'\cdot(x^3-8) - (x^2)(x^3-8)'}{(x^3-8)^2}$$

$$= \frac{2x(x^3-8) - x^2\cdot 3x^2}{(x^3-8)^2}$$

$$= \frac{2x^4-16x-3x^4}{(x^3-8)^2} = \underline{\frac{-16x-x^4}{(x^3-8)^2}}$$

Oppg. Deriver $g(x) = \frac{x^2-1}{x^3+4}$

La $f(x) = x^2-1$

(3) $h(x) = x^3+4$ $f'(x) = 2x$, $h'(x) = 3x^2$

$g(x) = \frac{f(x)}{h(x)}$ Kvotientregelen $g'(x) = \frac{f'(x)\cdot h(x) - f(x)\,h'(x)}{h^2(x)}$

$$g'(x) = \frac{(2x)(x^3+4) - (x^2-1)(3x^2)}{(x^3+4)^2}$$

$$= \frac{x(2x^3+8 - 3x(x^2-1))}{(x^3+4)^2}$$

$$= \frac{x(-x^3+3x+8)}{(x^3+4)^2}$$

Deriver $\frac{x^2+3x-4}{2x^3-4x+3}$

La $a(x) = x^2+3x-4$ $a'(x) = 2x+3$

$b(x) = 2x^3-4x+3$ $b'(x) = 6x^2-4$

$$\left(\frac{a(x)}{b(x)}\right)' = \frac{a'(x)\,b(x) - a(x)\,b'(x)}{b^2(x)}$$

$$= \frac{(2x+3)(2x^3-4x+3) - (x^2+3x-4)(6x^2-4)}{b^2(x)}$$

$$= \frac{(4x^4-8x^2+6x+6x^3-12x+9)}{b^2(x)}$$

$$- \frac{(6x^4+18x^3-24x^2-4x^2-12x+16)}{b^2(x)}$$

$$= \frac{-2x^4-12x^3+20x^2+6x-7}{(2x^3-4x+3)^2}$$

Deriver $\dfrac{\sqrt{x}}{x^2-3}$ (Hint $\sqrt{x} = x^{1/2}$)

(4)

$$\left(\frac{\sqrt{x}}{x^2-3}\right)' = \frac{(\sqrt{x})'(x^2-3) - \sqrt{x}\,(x^2-3)'}{(x^2-3)^2}$$

$$= \frac{\left(\frac{1}{2}\cdot x^{-1/2}\right)(x^2-3) - \sqrt{x}\,(2x)}{(x^2-3)^2} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{\left(\frac{1}{2}\cdot 2\cdot \frac{1}{\cancel{\sqrt{x}}}\cdot\sqrt{x}\right)(x^2-3) - (2\sqrt{x}\cdot\sqrt{x})\cdot 2x}{(x^2-3)^2\cdot 2\sqrt{x}}$$

$$= \frac{x^2-3 - 2x\cdot 2x}{(x^2-3)^2\cdot 2\sqrt{x}}$$

$$= \frac{-3x^2-3}{(x^2-3)^2\, 2\sqrt{x}} = \underline{\underline{\frac{-3(x^2+1)}{2\sqrt{x}\,(x^2-3)^2}}}$$

Deriver $f(x) = \dfrac{\sqrt{x}}{(x^2-3)^5} = \sqrt{x}\cdot(x^2-3)^{-5}$

$$f'(x) = \left(\sqrt{x}\cdot(x^2-3)^{-5}\right)' = (\sqrt{x})'(x^2-3)^{-5} + \sqrt{x}\cdot\left((x^2-3)^{-5}\right)'$$

$$= \left(\frac{1}{2}\cdot x^{-1/2}\right)(x^2-3)^{-5} + \sqrt{x}\left[\underbrace{(-5)(x^2-3)^{-5-1}\cdot(x^2-3)'}_{-5(x^2-3)^{-6}\cdot 2x}\right]$$

$$= \frac{1}{2}x^{-1/2}(x^2-3)^{-5} + (-10)x\sqrt{x}\,(x^2-3)^{-6}$$

$$= \frac{1}{2\sqrt{x}\,(x^2-3)^5}\cdot\frac{x^2-3}{x^2-3} + \frac{-10x\sqrt{x}}{(x^2-3)^6}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{(x^2-3) - 20x^2}{2\sqrt{x}\,(x^2-3)^6} = \underline{\frac{-19x^2-3}{2\sqrt{x}\,(x^2-3)^6}}$$

La $f(x) = \frac{x}{x^2+1}$

5. Finn topp/bunn punkt
- vende punkt

Skiser grafen.

$$f'(x) = \frac{(x)'(x^2+1) - x(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{(x^2+1) - 2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}.$$

Kritiske punkt.

$f'(x) = 0$ når $1-x^2 = (1-x)(1+x) = 0$

$x=-1$ og $x=1$.

$f'(x) > 0$ for $x \in (-1, 1)$

$f'(x) < 0$ for $|x| > 1$.

bunnpunkt : $(-1, f(-1)) = (-1, -\frac{1}{2})$

toppunkt : $(1, f(1)) = (1, \frac{1}{2})$.

$$f''(x) = (f'(x))' = \left(\frac{1-x^2}{(x^2+1)^2}\right)' = \left((1-x^2)\cdot(x^2+1)^{-2}\right)'$$

$$= (-2x)(x^2+1)^{-2} + (1-x^2)\cdot\left(-2(x^2+1)^{-3}\right)\cdot(x^2+1)'$$

$$= \frac{-2x}{(x^2+1)^2}\cdot\frac{x^2+1}{x^2+1} + \frac{(-2)2x(1-x^2)}{(x^2+1)^3} = \frac{-2x(x^2+1) - 4x(1-x^2)}{(x^2+1)^3}$$

$$= \frac{2x^3-6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}.$$

$f''(x) = 0$ når $x = -\sqrt{3}$, 0 og $\sqrt{3}$.

$f''(x)$ skifter fortegn rundt hver av disse verdiene

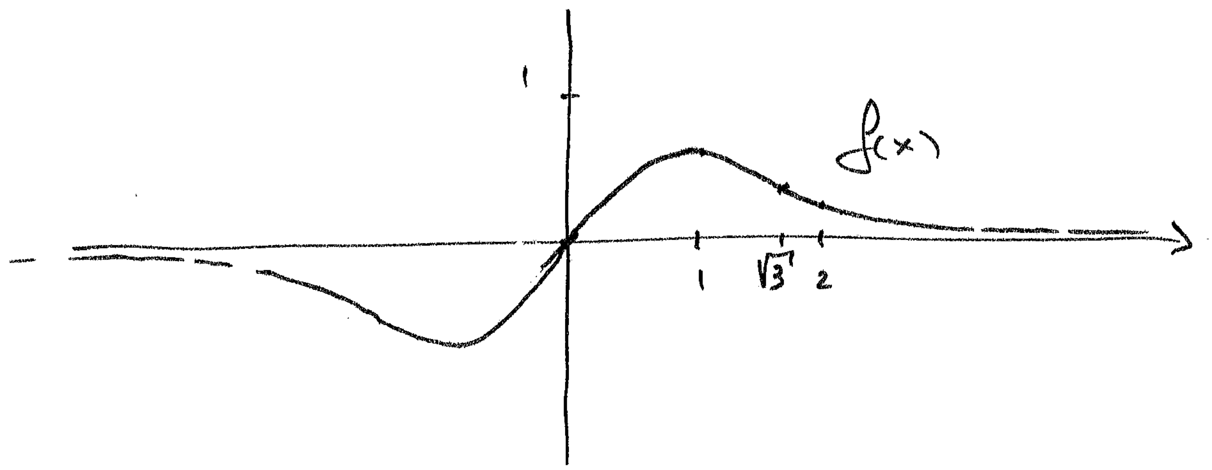
Vendepunktene er:

⑥

$(0,0)$, $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$ $\left(f(-\sqrt{3})\right)$

$\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

$\sqrt{3} \sim 1.73$



symmetrisk om origo